The magic of math: three-dimensional X-ray vision

Samuli Siltanen

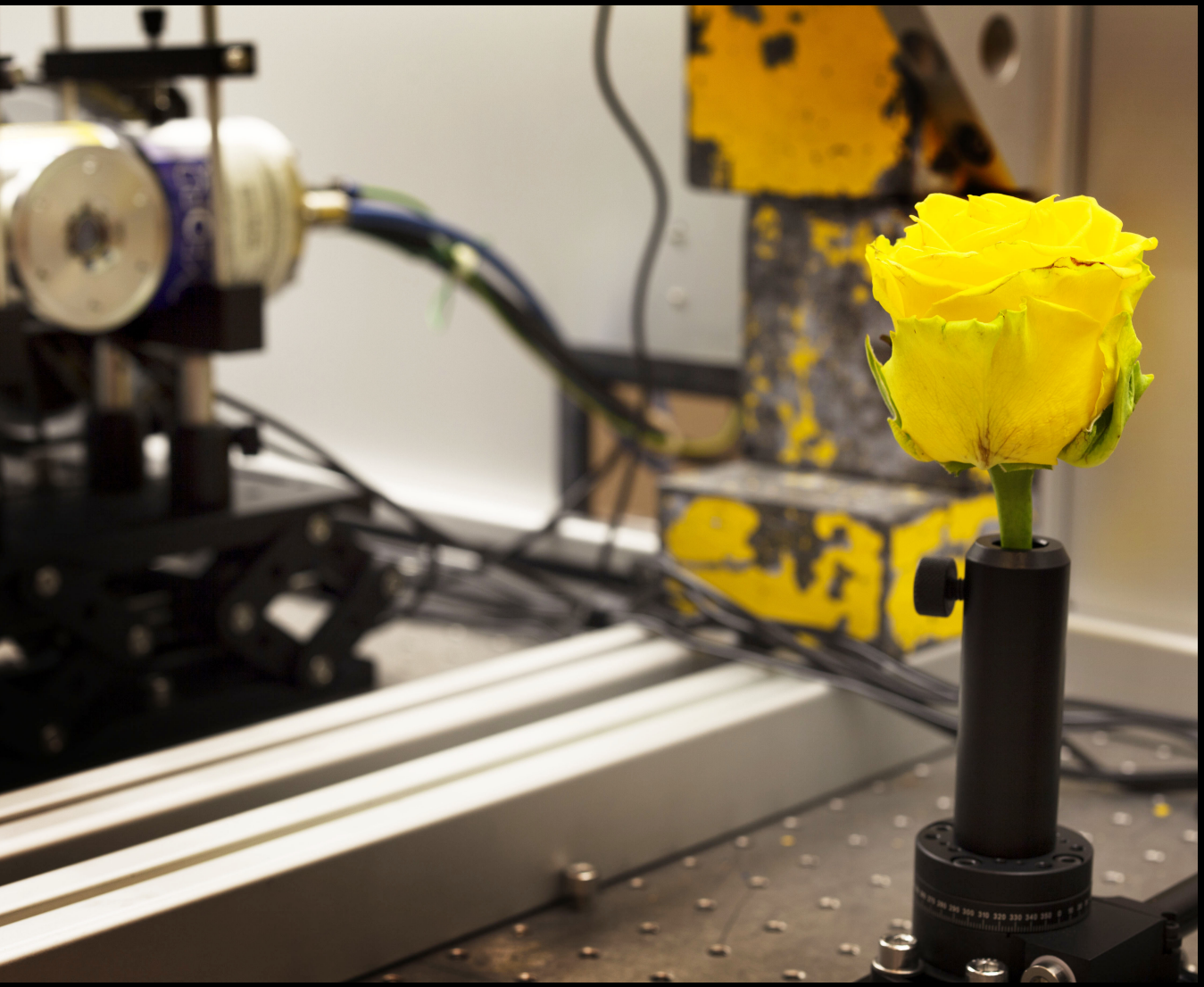
Department of Mathematics and Statistics
University of Helsinki, Finland
samuli.siltanen@helsinki.fi
www.siltanen-research.net

Coloquio MTM UFSC
Web-talk
March 19, 2021

Outline

We can see through a box of candy!

https://www.dropbox.com/s/e7i3exqc4sdpr1s/Sisu2.mp4?dl=0




HAMAMATSU
DIGITAL OUT
EXT TRG
POWER

X-ray images are very useful for doctors. For example, they can see fractures.

Nevit Dilmen, Wikimedia commons

Outline

Here is a 2D slice through a human head

Andrew Ciscel, Wikimedia commons

After calibration we are observing how much attenuating matter the X-ray encounters in total

https://youtu.be/RFArLtWEfsQ

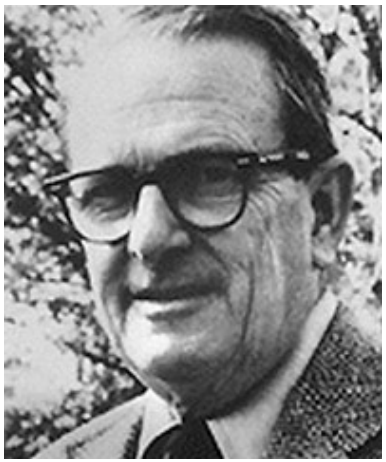
This sweeping movement is the data collection mode of first-generation CT scanners

https://youtu.be/JHUz5oyeZb0

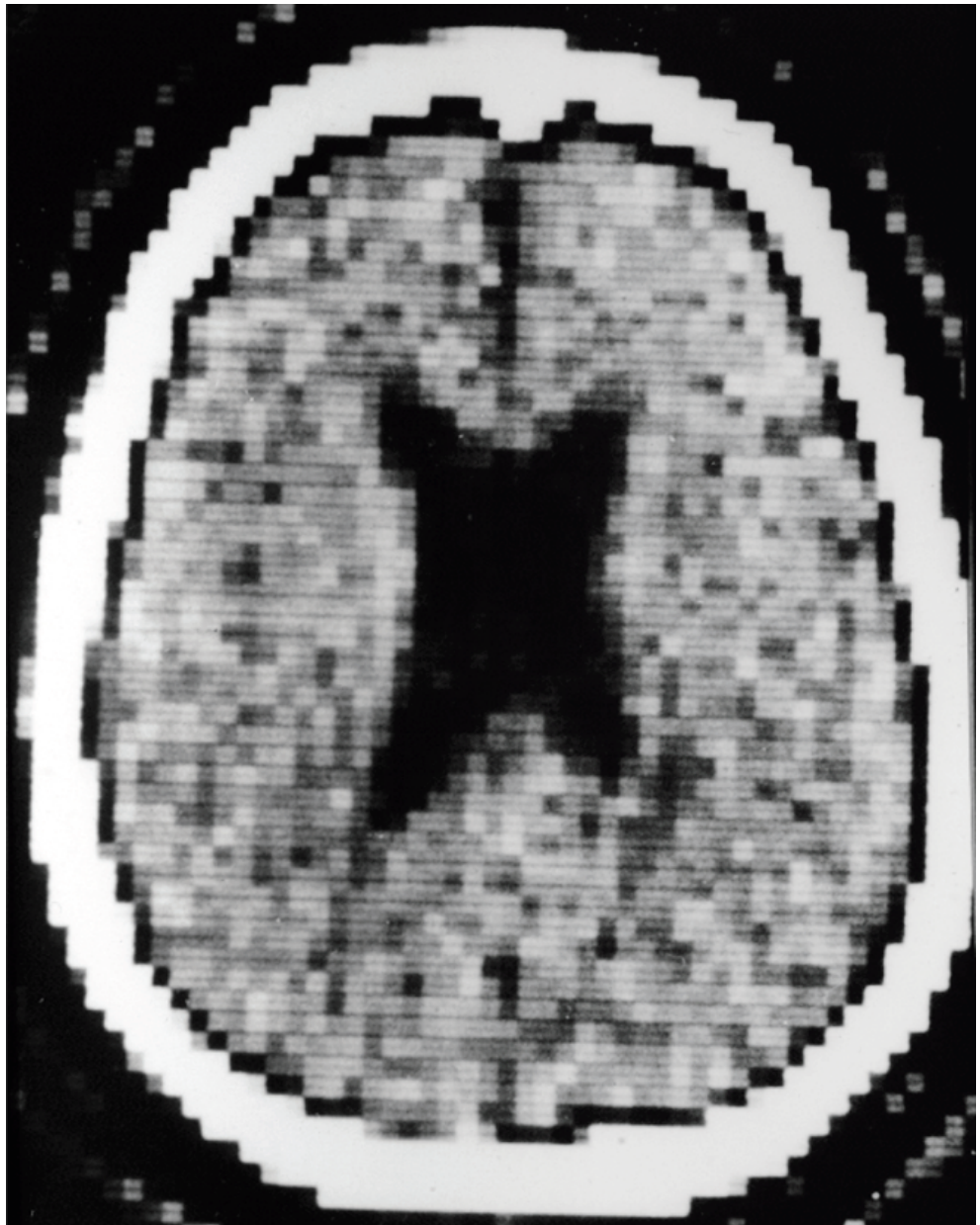
Data is collected by rotating the system around the patient

https://youtu.be/newxZbw7YAs

Godfrey Hounsfield and Allan McLeod Cormack developed X-ray tomography



Hounsfield (top) and Cormack received Nobel prizes in 1979.




Couch unit for EMI brain scanner.
Image: Science Museum Group.




Modern CT scanners look like this

Wikimedia commons

Modern scanners rotate at high speed

https://commons.wikimedia.org/wiki/File:CT-Rotation.ogv

This is the inverse problem of tomography: we only know the data

https://youtu.be/pr8bXB0oAqI

This is an illustration of the standard reconstruction by filtered back-projection

https://youtu.be/tRD58lO1FKw

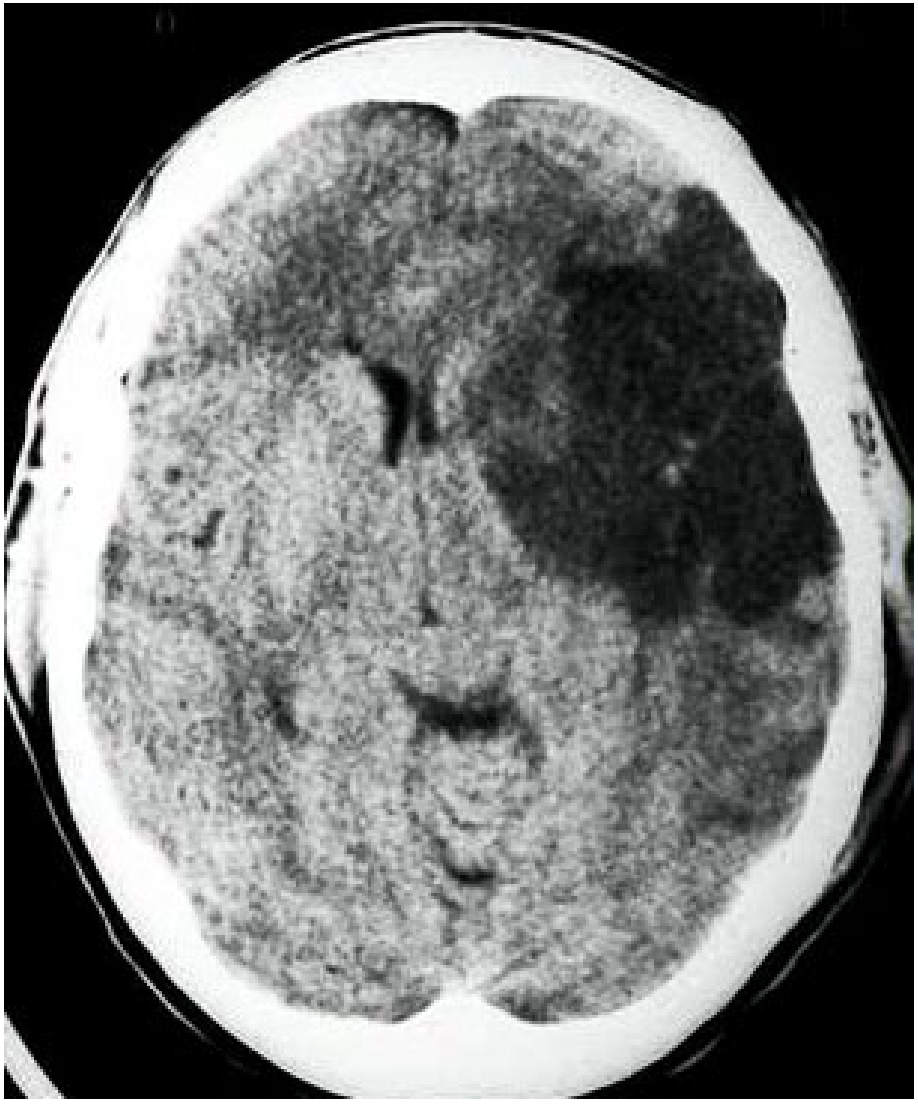
Reconstruction of a function from its line integrals was first invented by Johann Radon in 1917

Johann Radon (1887-1956)

$$f(P) = -\frac{1}{\pi}\int_0^\infty \frac{d\overline{F_p}(q)}{q}$$

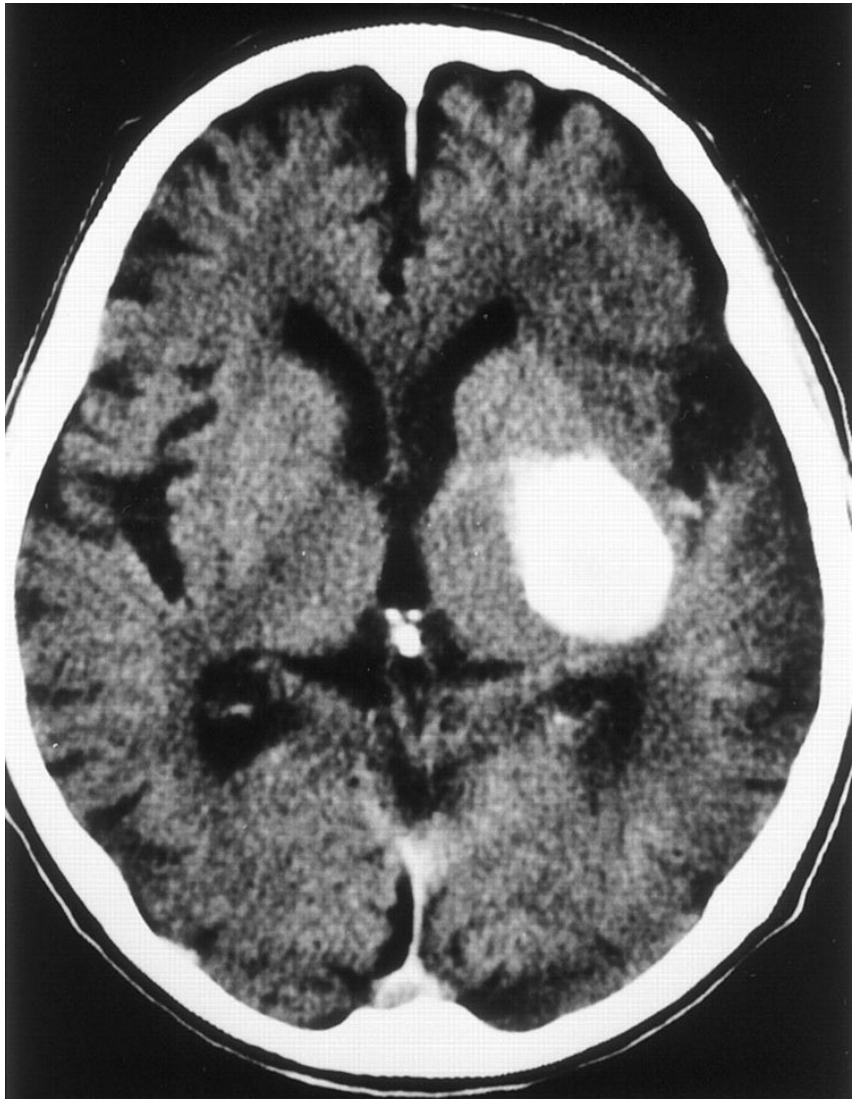
Diagnosing stroke with X-ray tomography

Ischemic stroke



CT image from Jansen 2008

Hemorrhagic stroke



CT image from Nakano *et al.* 2001

Unusual variant of the Nutcracker Fracture of the calcaneus and tarsal navicular

[Gajendran, Yoo & Hunter, Radiology Case Reports **3** (2008)]

Outline

Let's warm up before the tests.
Here is tomographic data of a simple object:

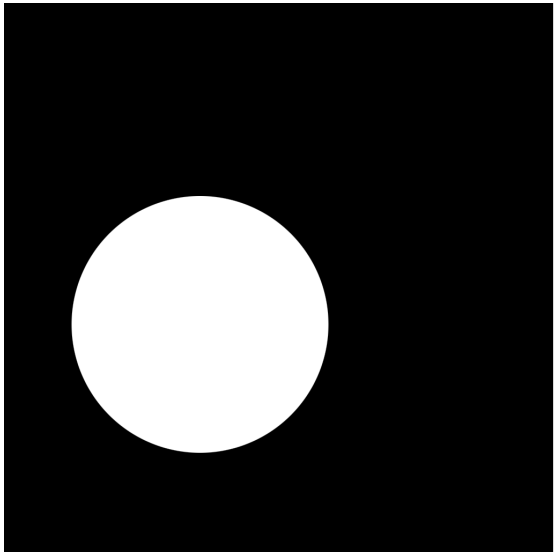
Can you guess the shape of the object from the tomographic data?

Test: can you guess the image?

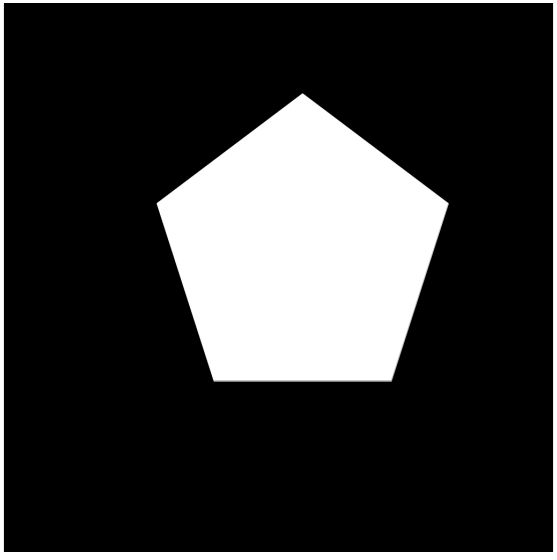
https://youtu.be/NishyJWhXDk

Alternatives

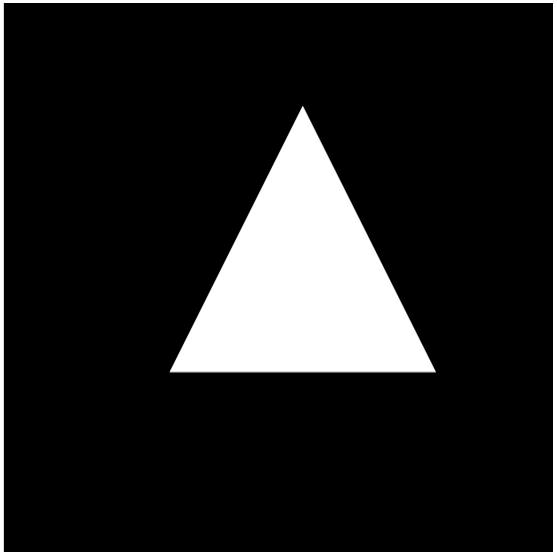
(a)



(b)



(c)



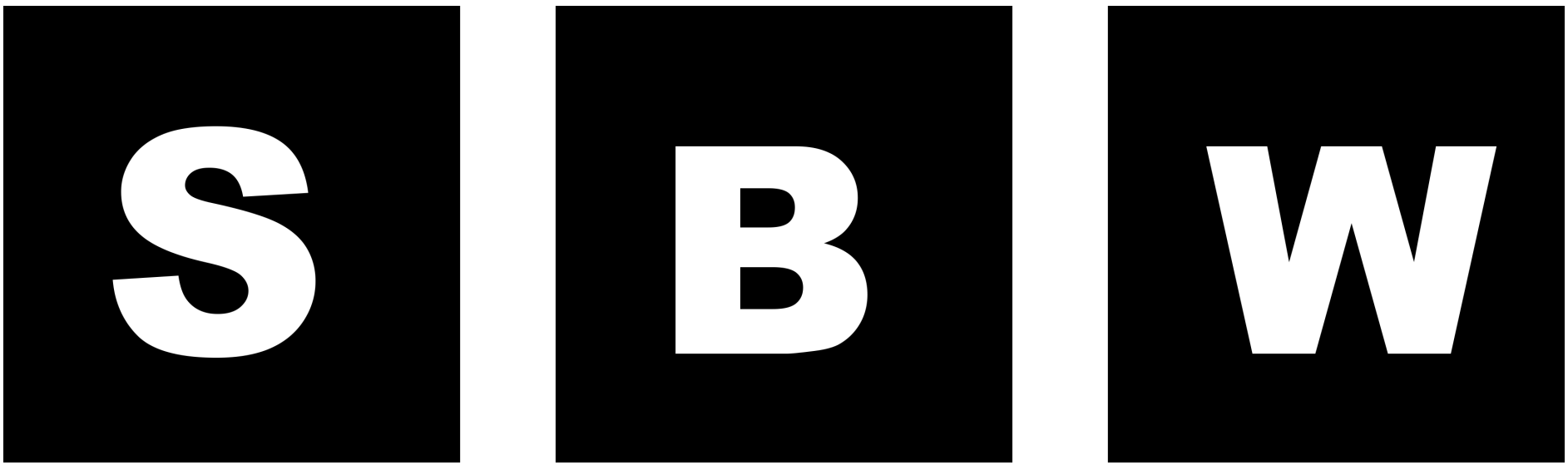
Solution

https://youtu.be/MkAQoF3YOwg

Test: can you guess the image?

https://youtu.be/ZJaek4nkcRA

Alternatives



Solution

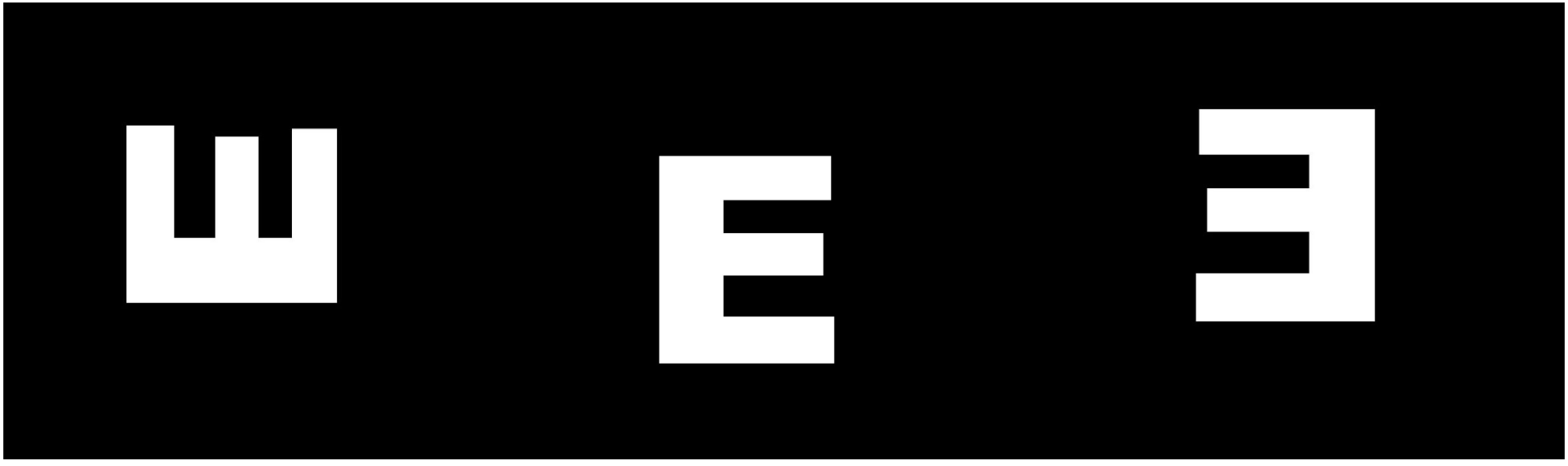
https://youtu.be/YHpG5HqDmZk

Test: can you guess the image?

https://youtu.be/RW2zso9WayI

Alternatives

(a) (b) (c)



Solution

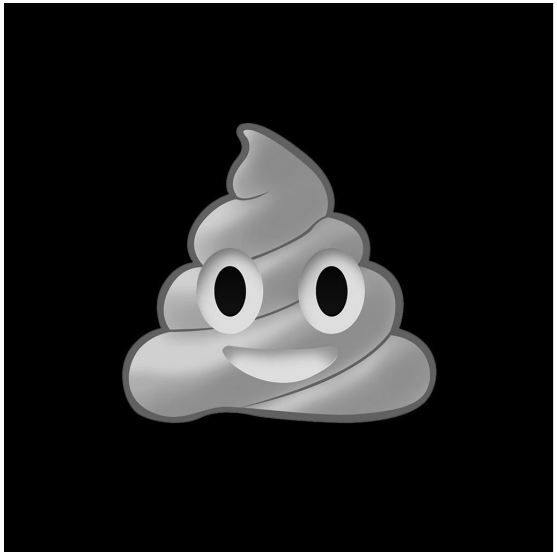
https://youtu.be/k0ArBgCx0n0

Test: can you guess the image?

https://youtu.be/goddXsubZO8

Alternatives

(a)



(b)

(c)

Solution

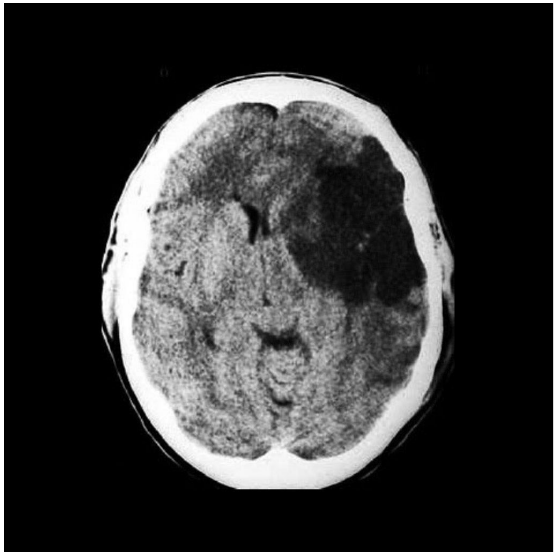
https://youtu.be/RfKA3R2-pjk

Test: can you guess the image?

https://youtu.be/8ZrRazVdRjM

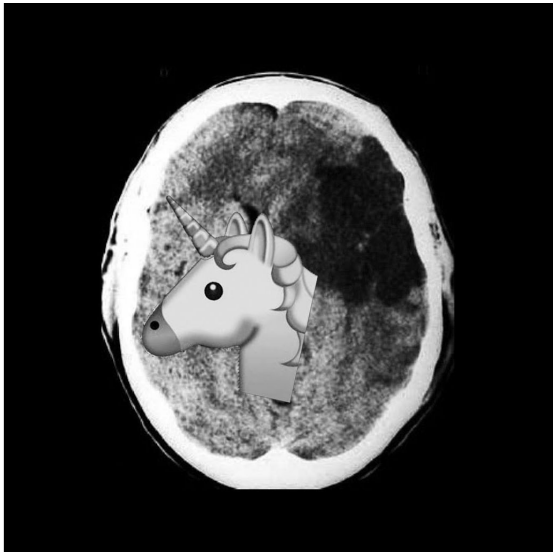
Alternatives

(a)



(b)

(c)



Solution

https://youtu.be/vLdQMDbptjM

https://youtu.be/EAQcMB-0cVo

Outline

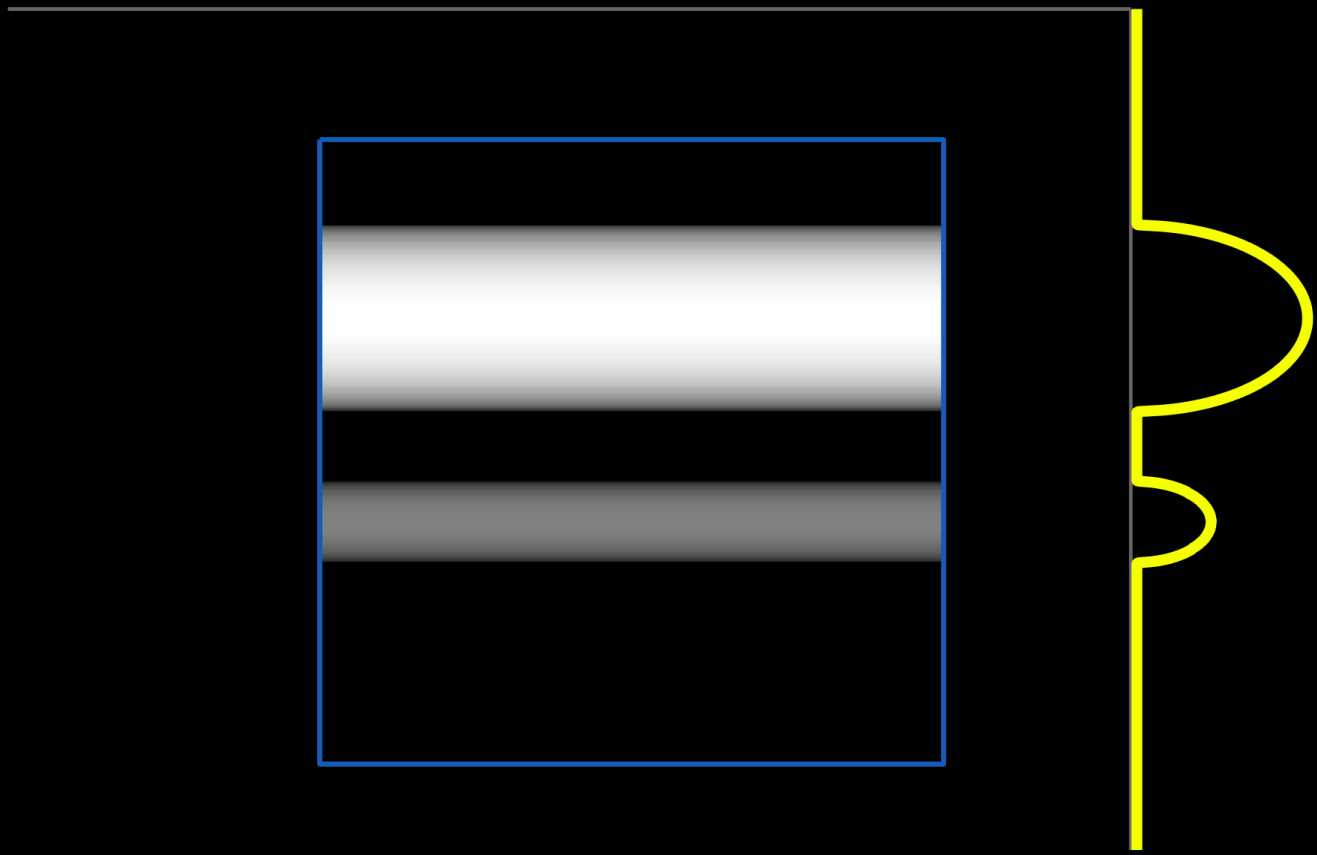
Here is a simple example of tomographic data collection, with two discs as the target

https://youtu.be/5DUGTXd26nA

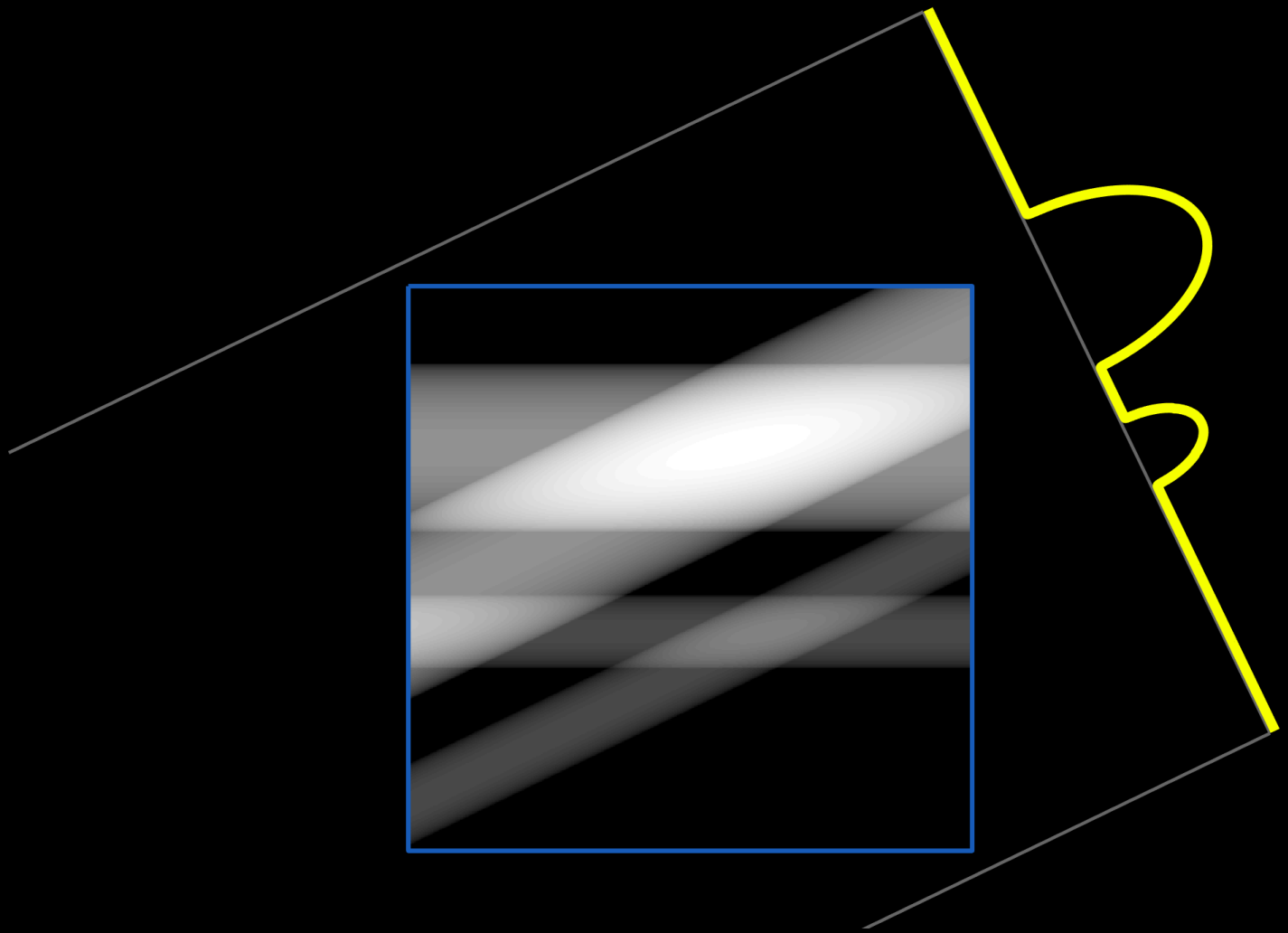
The inverse problem of tomography is to recover the unknown target from the measured X-ray data

https://youtu.be/YhCIb0MaB70

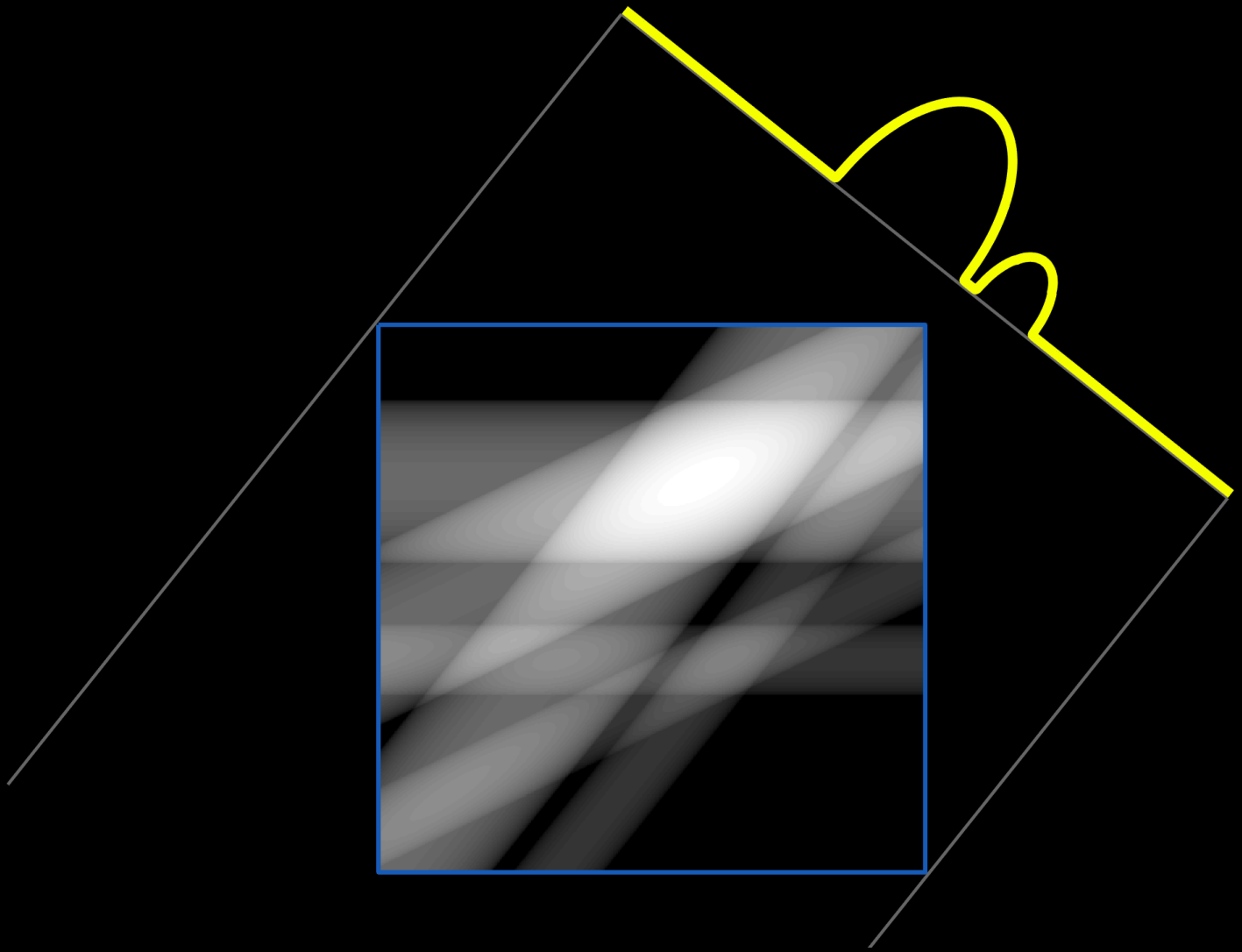
Summing up all the back-projections results in a blurred reconstruction



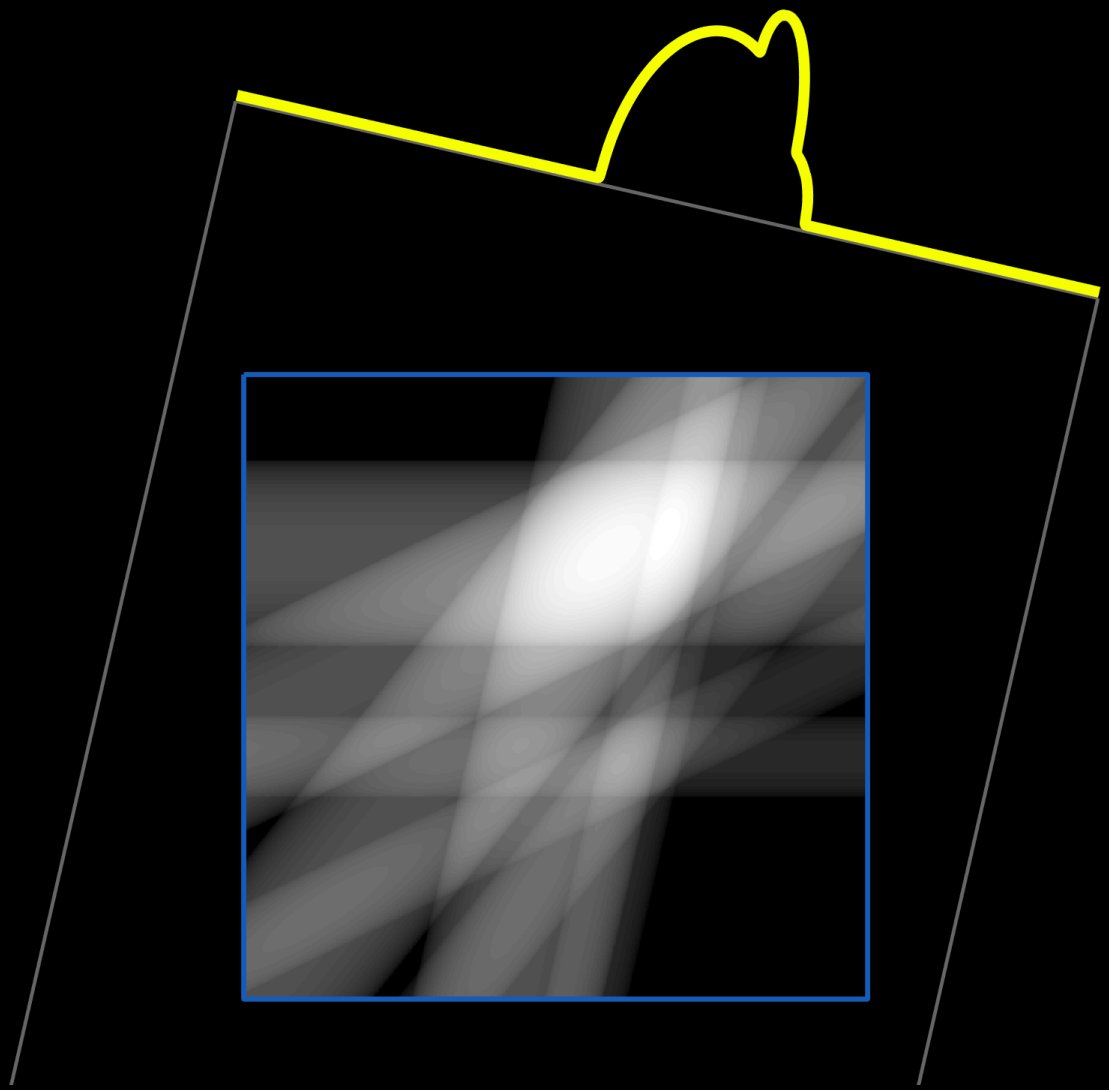
Summing up all the back-projections results in a blurred reconstruction



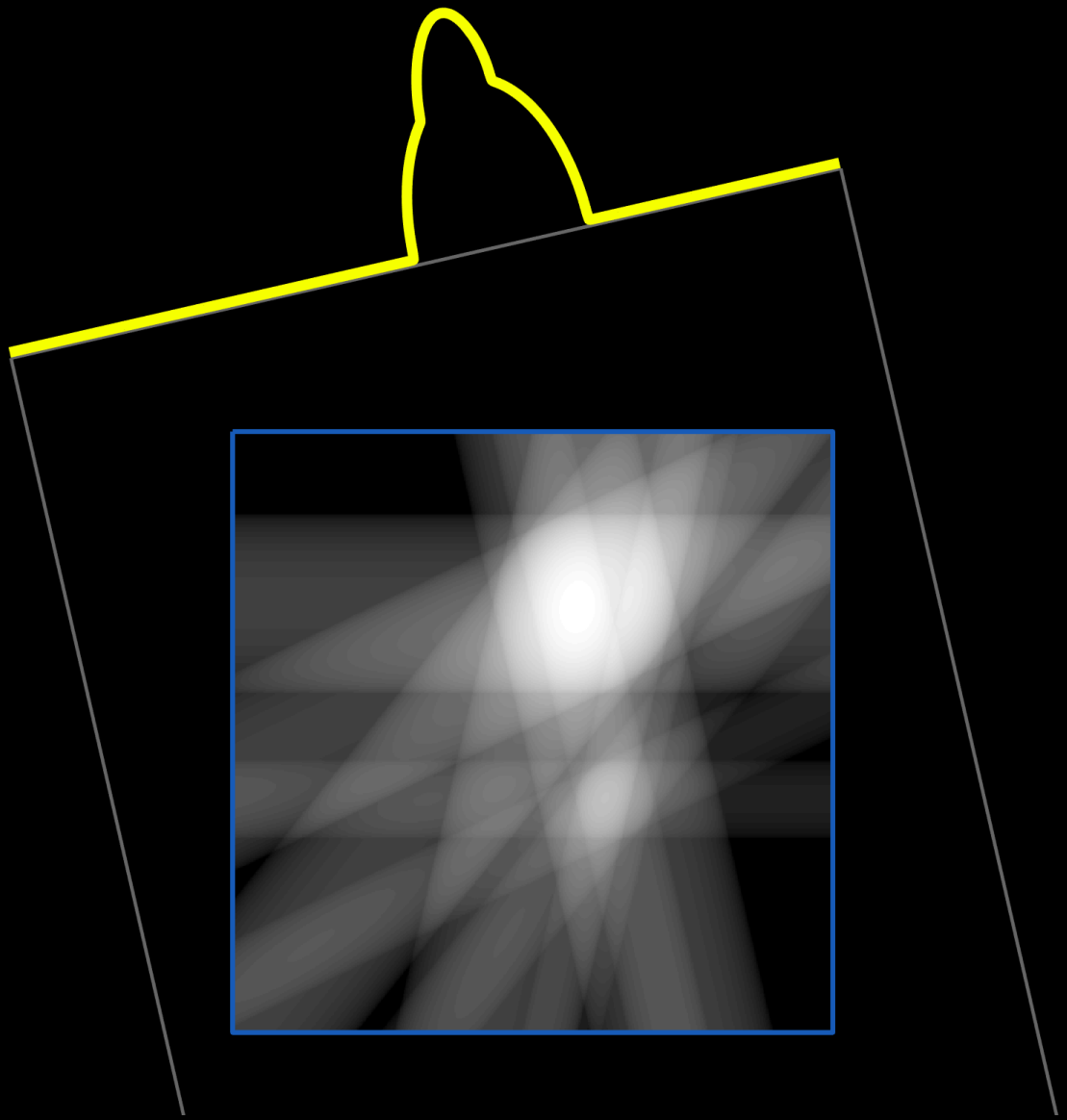
Summing up all the back-projections results in a blurred reconstruction



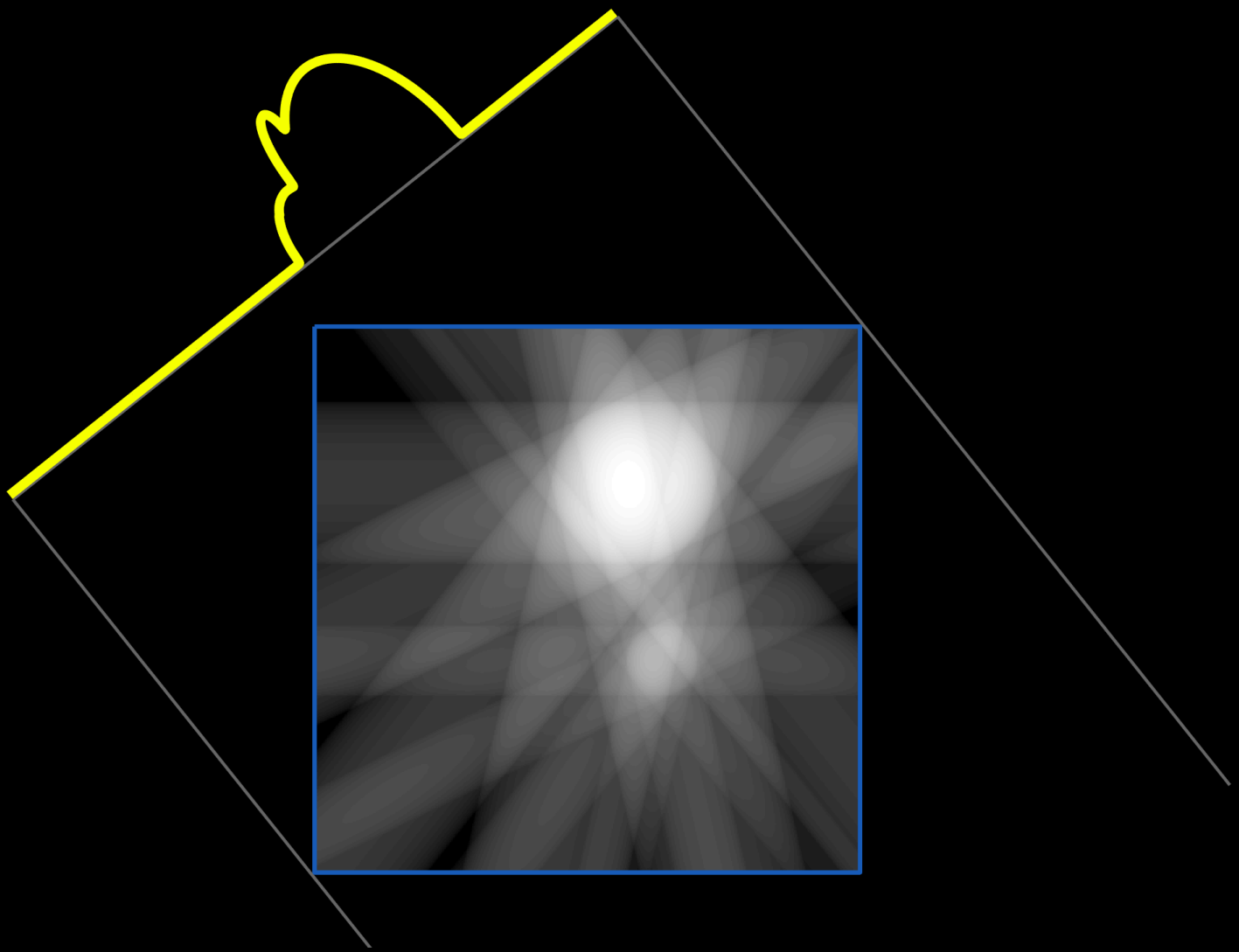
Summing up all the back-projections results in a blurred reconstruction



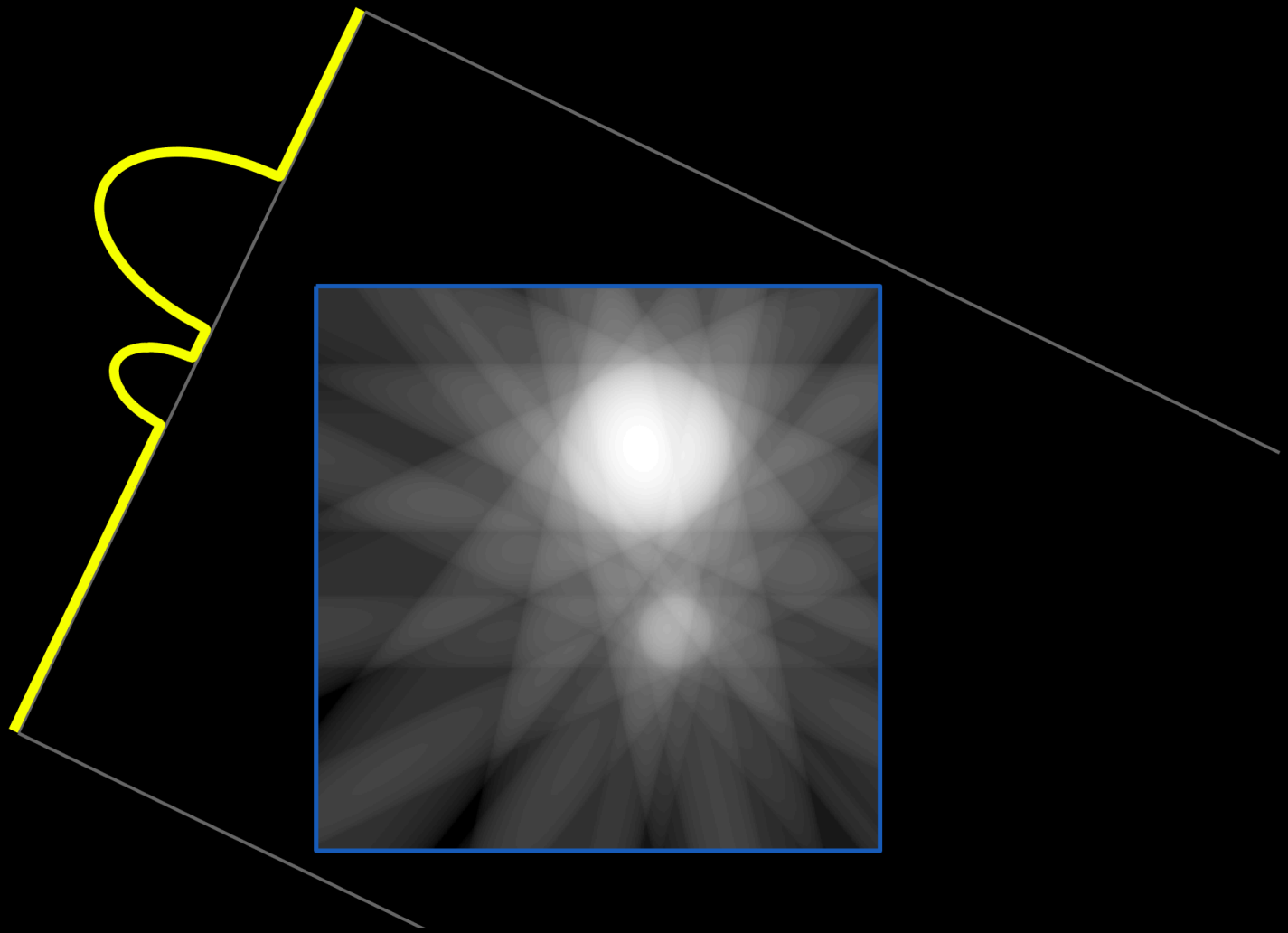
Summing up all the back-projections results in a blurred reconstruction



Summing up all the back-projections results in a blurred reconstruction



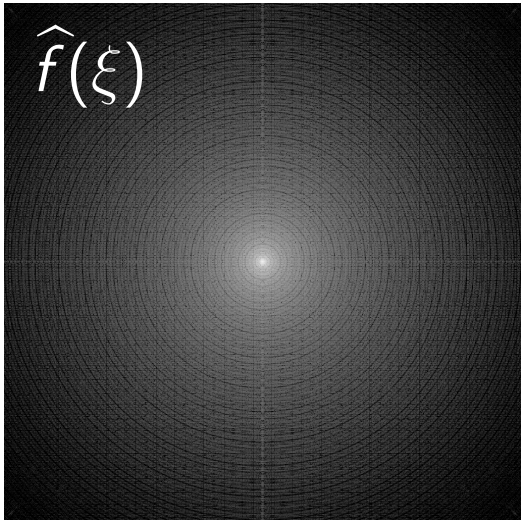
Summing up all the back-projections results in a blurred reconstruction



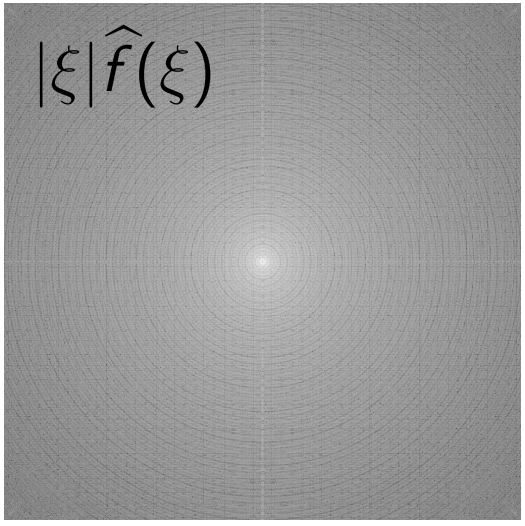
Here we use more directions, so the reconstruction quality is higher

https://youtu.be/5DUGTXd26nA

Final reconstruction involves high-pass filtering on top of the back-projection

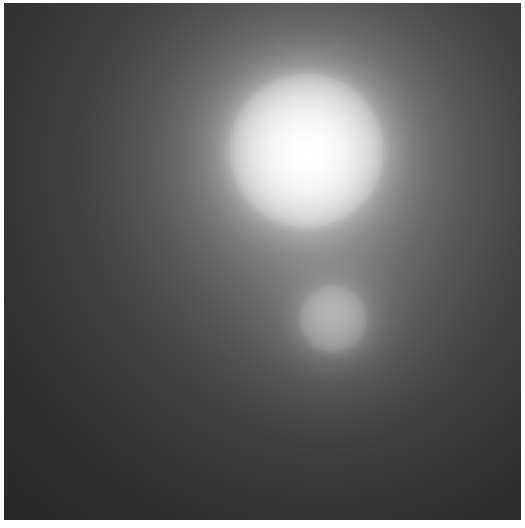


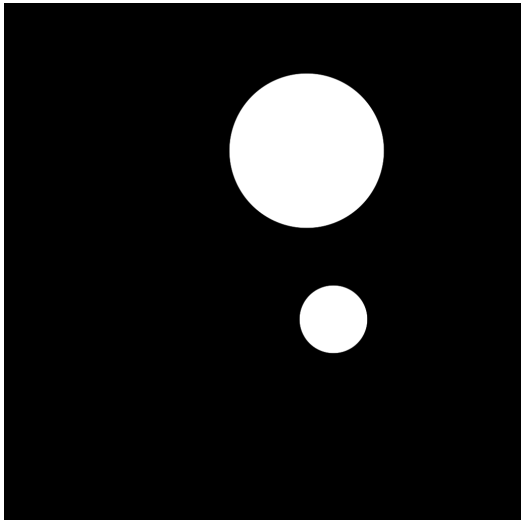
Boost high frequencies



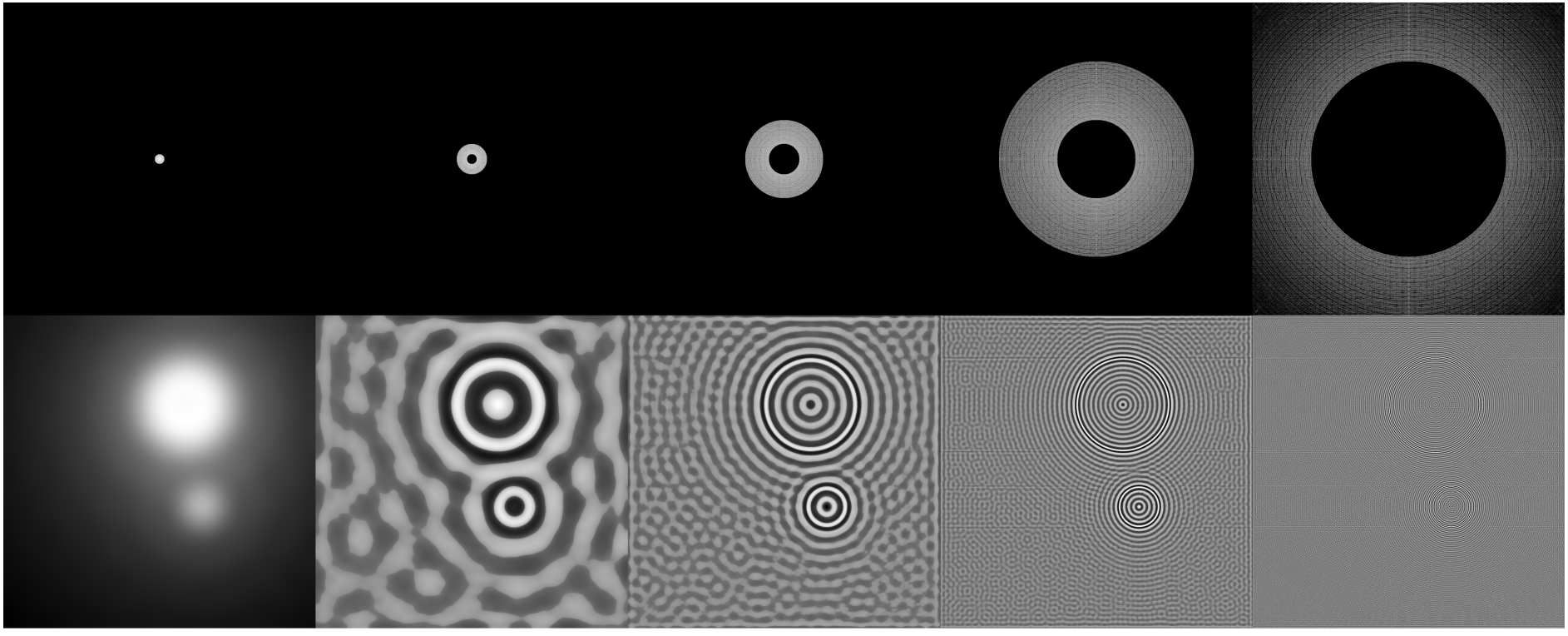
FFT

IFFT

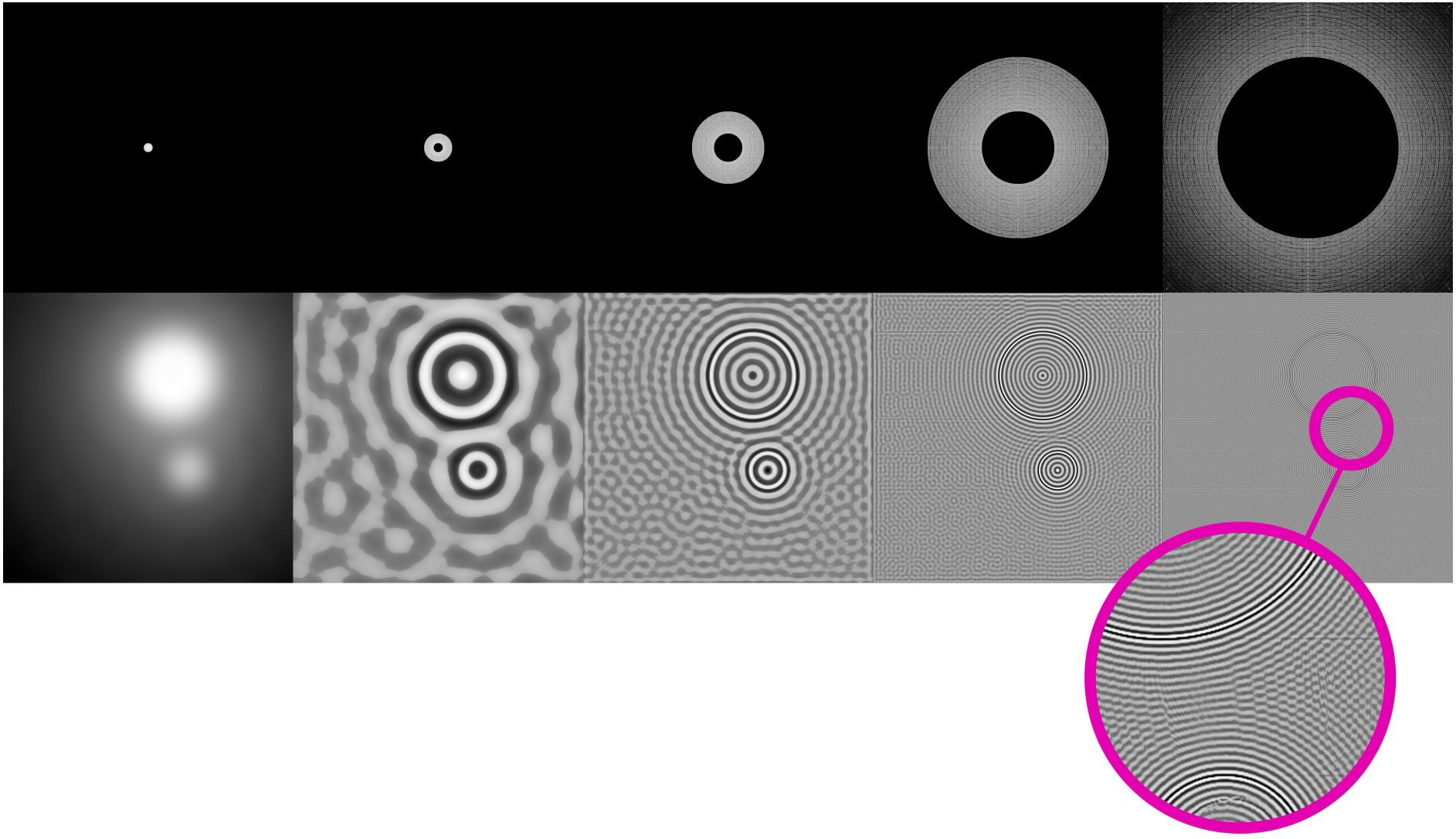




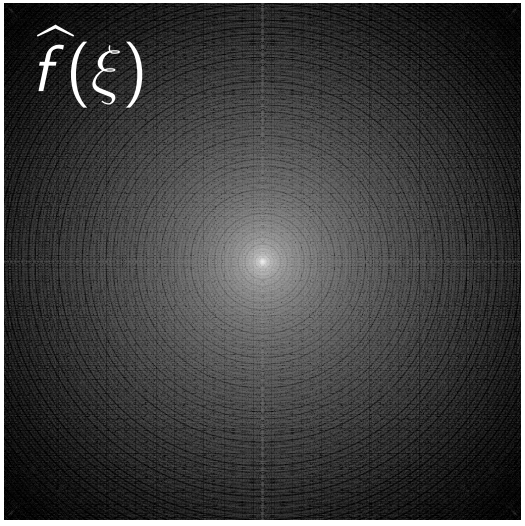
Let's observe the Fourier transform by dividing it into frequency bands



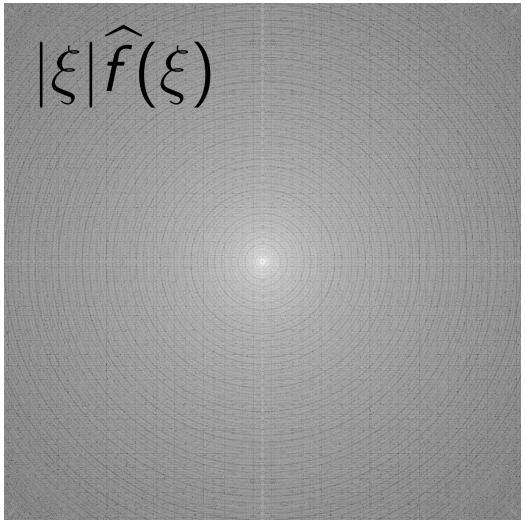
Let's observe the Fourier transform by dividing it into frequency bands



Final reconstruction involves high-pass filtering on top of the back-projection

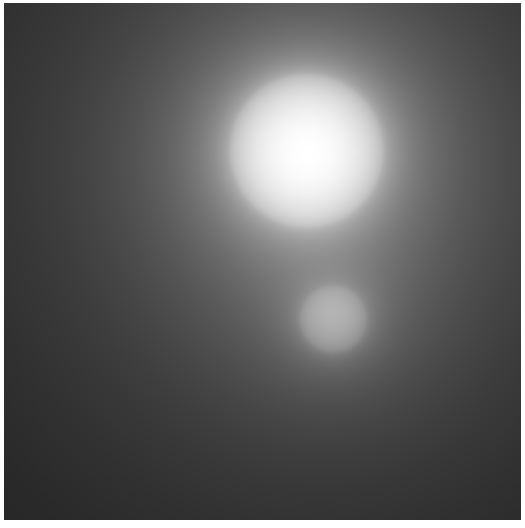


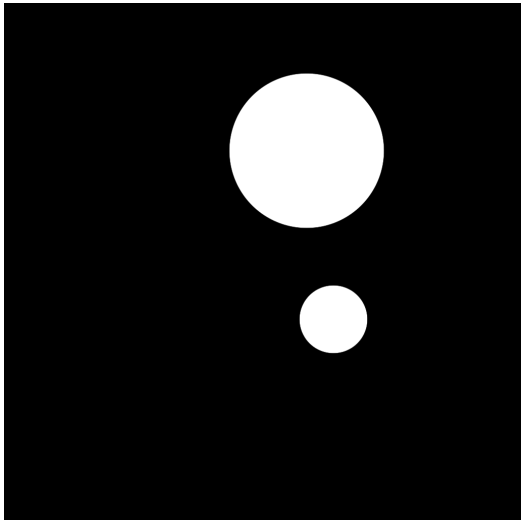
Boost high frequencies



FFT

IFFT



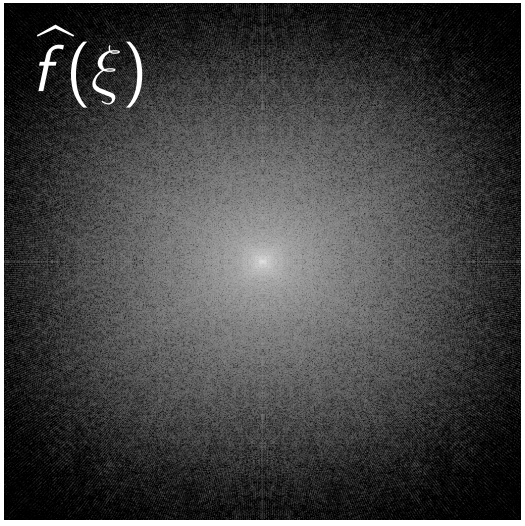



Fa So La
HELLO KITTY
COLORADO
STATE

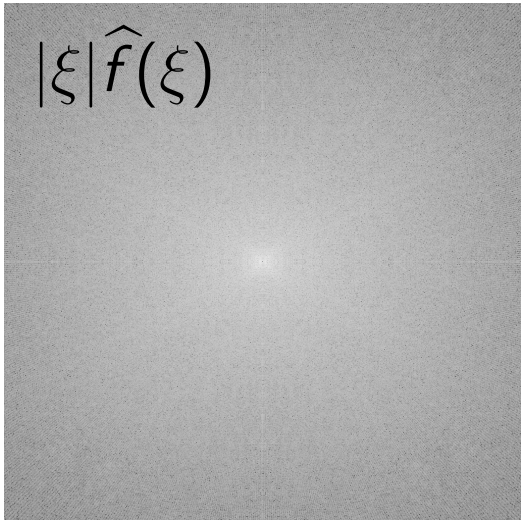

Fa So La
HELLO KITTY

The head slice image is reconstructed
by first applying unfiltered back-projection

Final reconstruction involves high-pass filtering on top of the back-projection

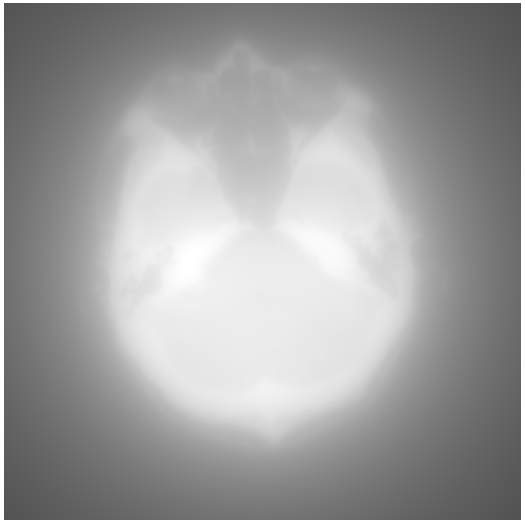


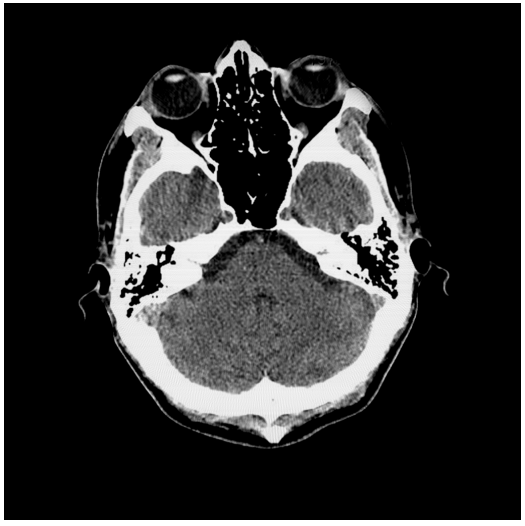
Boost high frequencies



FFT

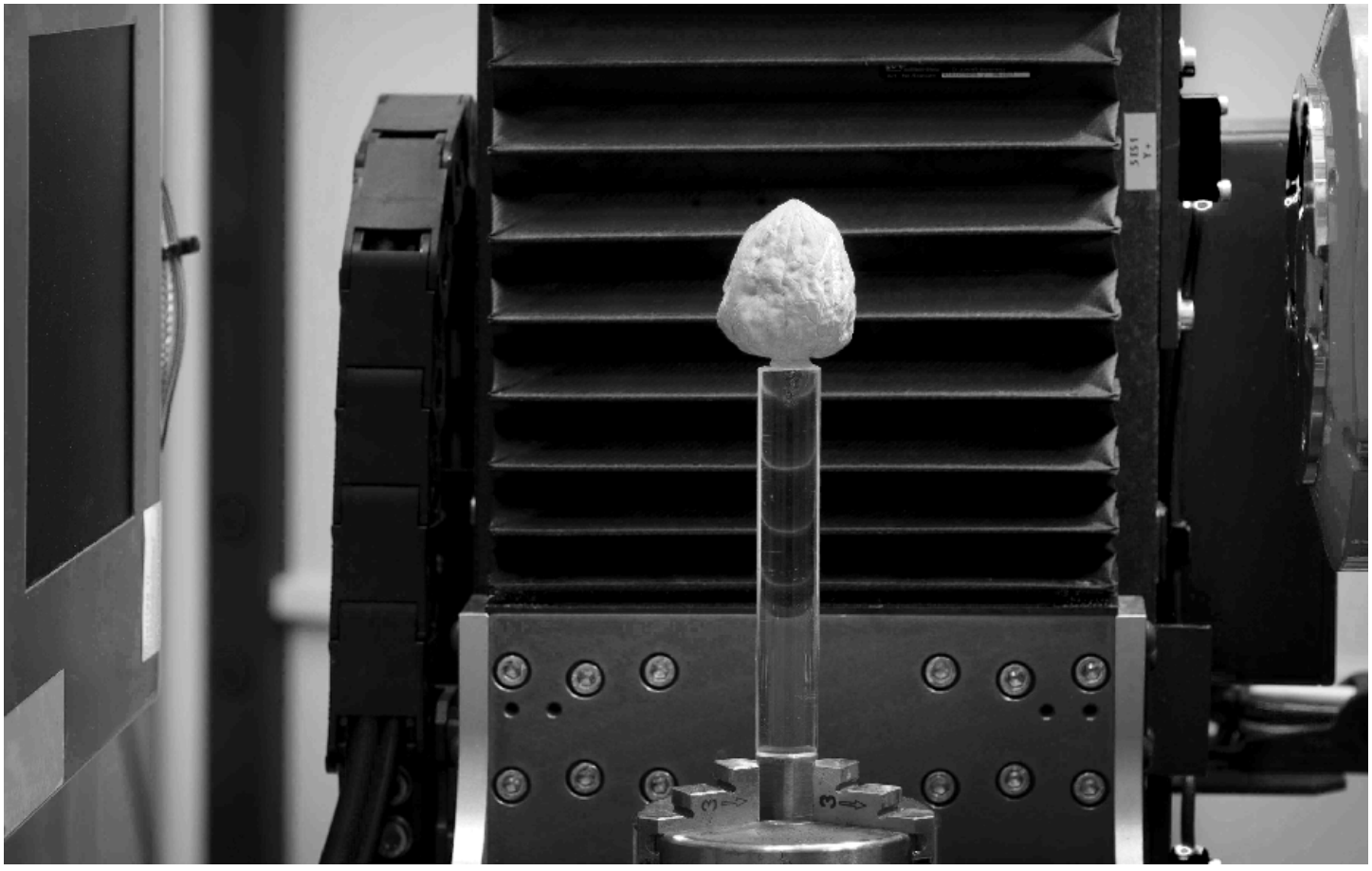
IFFT

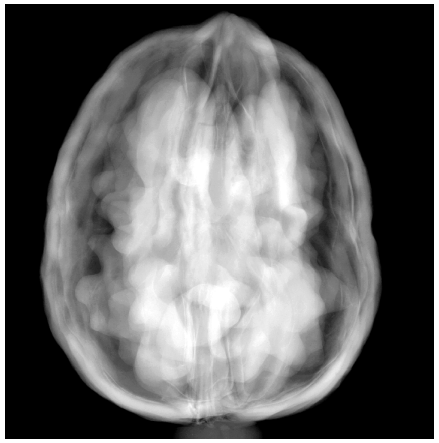


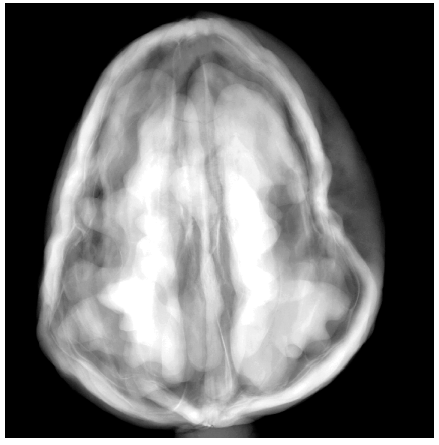


Outline

We collected X-ray projection data of a walnut from 1200 directions



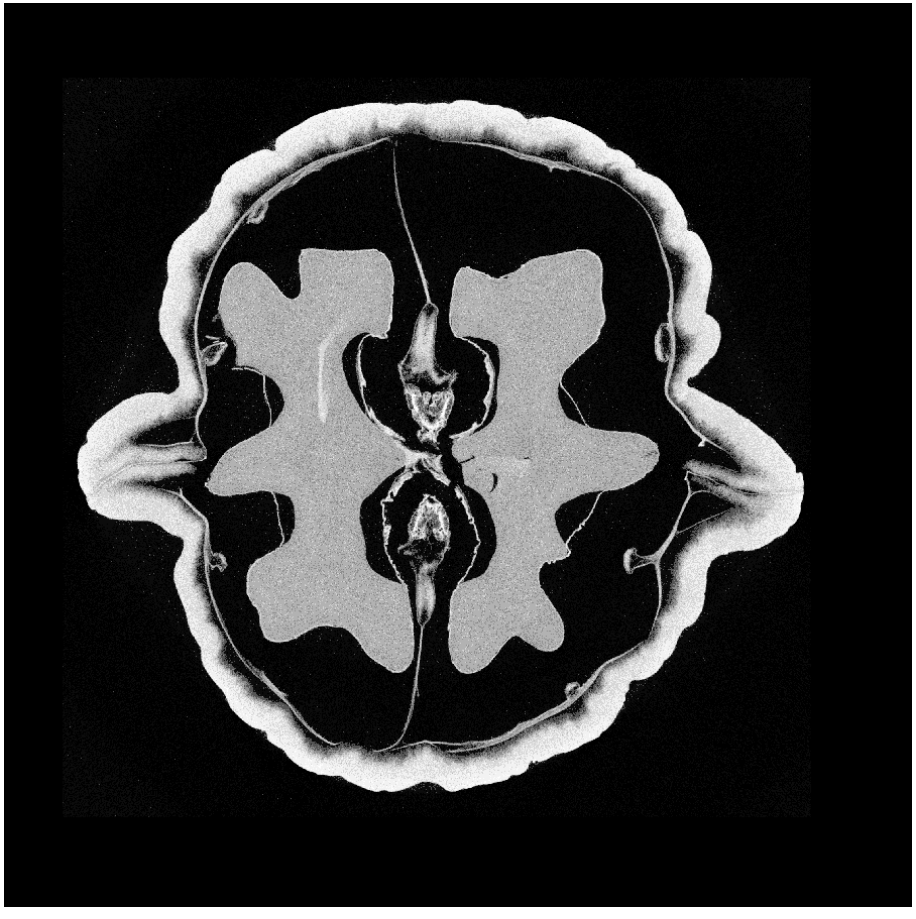




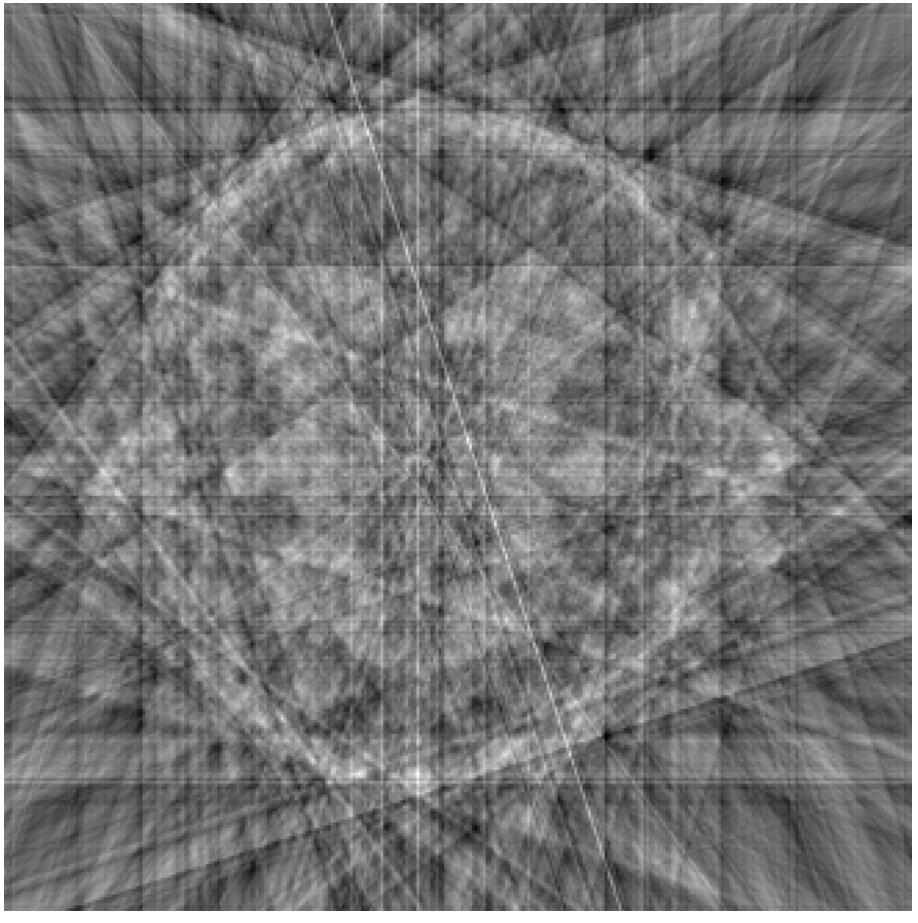
Data collection: thanks to Keijo Hämäläinen and Aki Kallonen, University of Helsinki.

The data is openly available at http://fips.fi/dataset.php, thanks to Esa Niemi and Antti Kujanpää

Reconstructions of a 2D slice through the walnut using filtered back-projection (FBP)

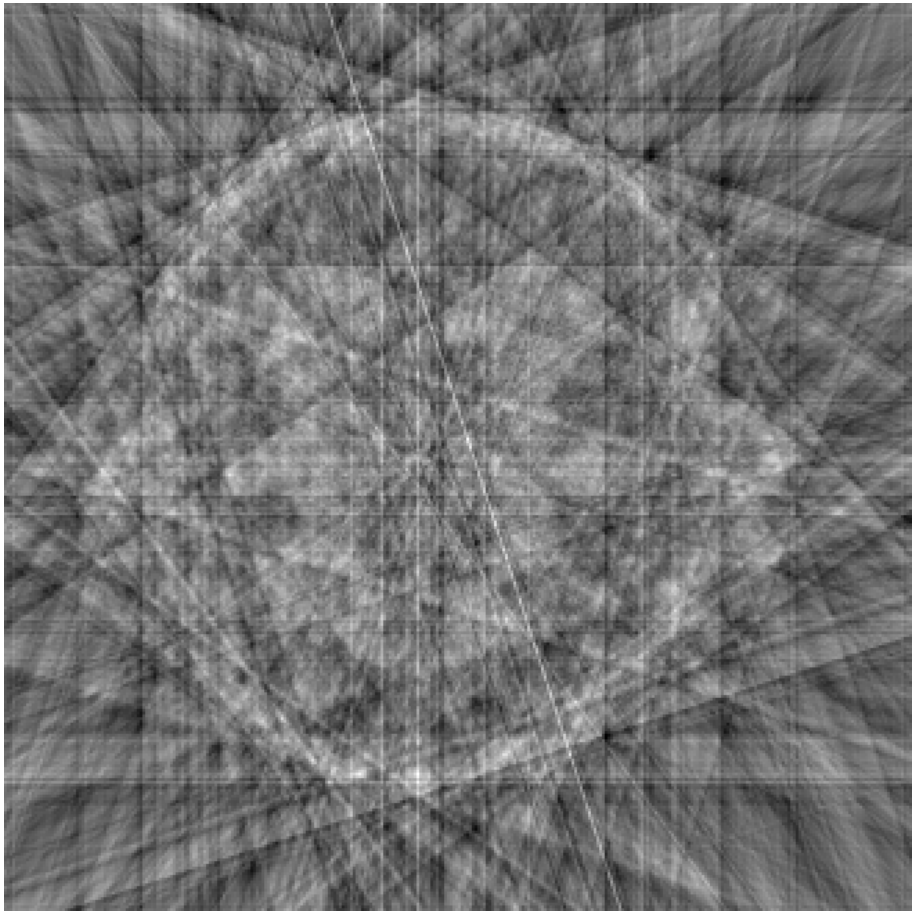


FBP with comprehensive data
(1200 projections)

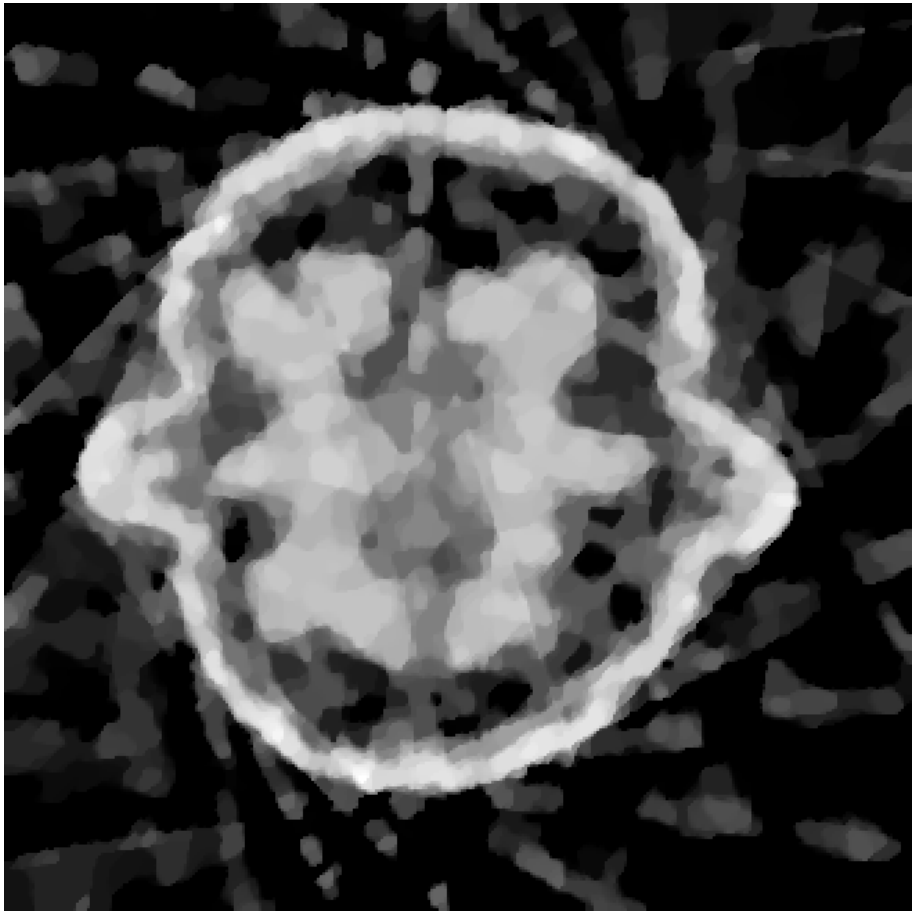


FBP with sparse data
(20 projections)

Sparse-data reconstruction of the walnut using non-negative total variation regularization

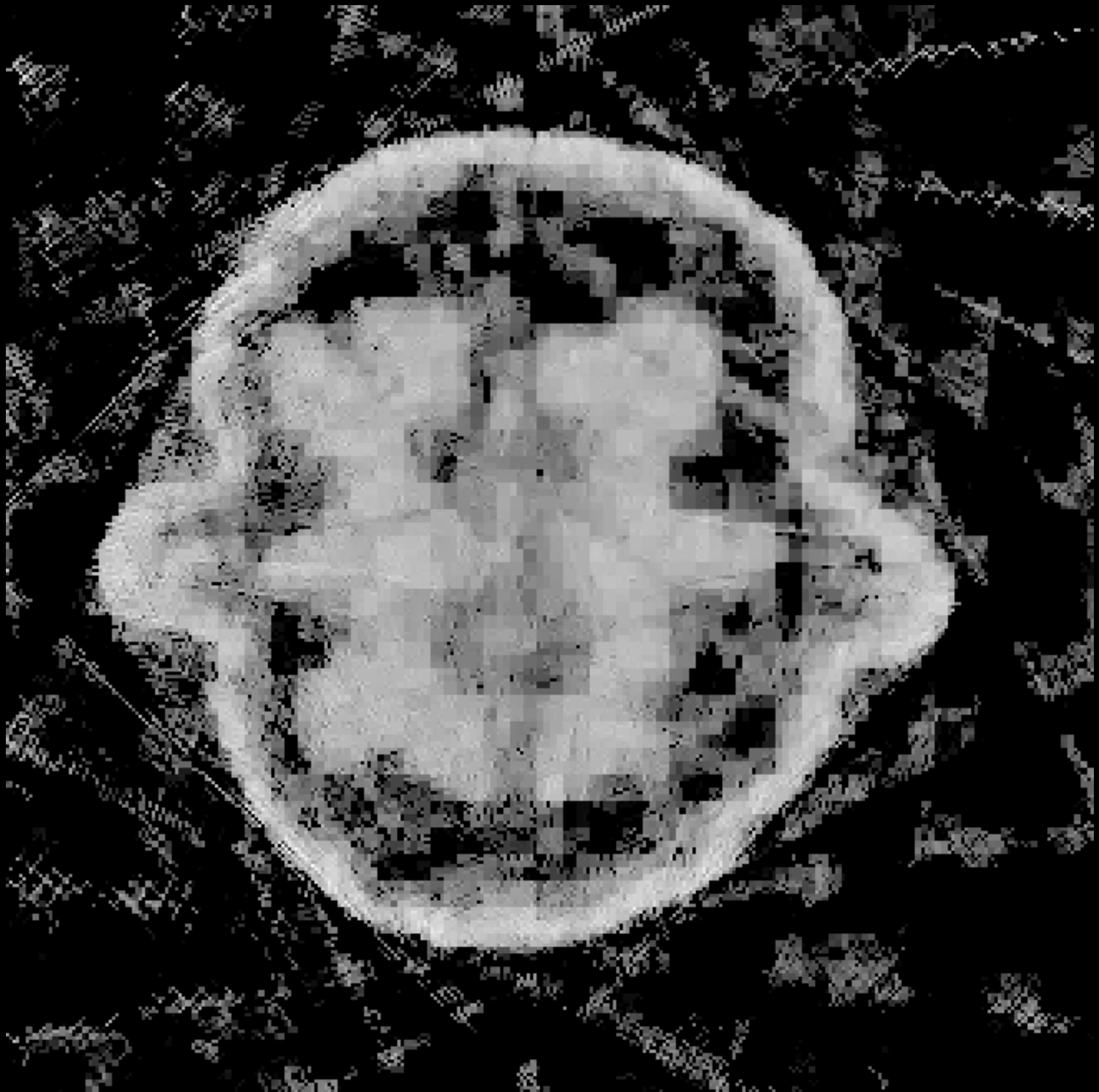


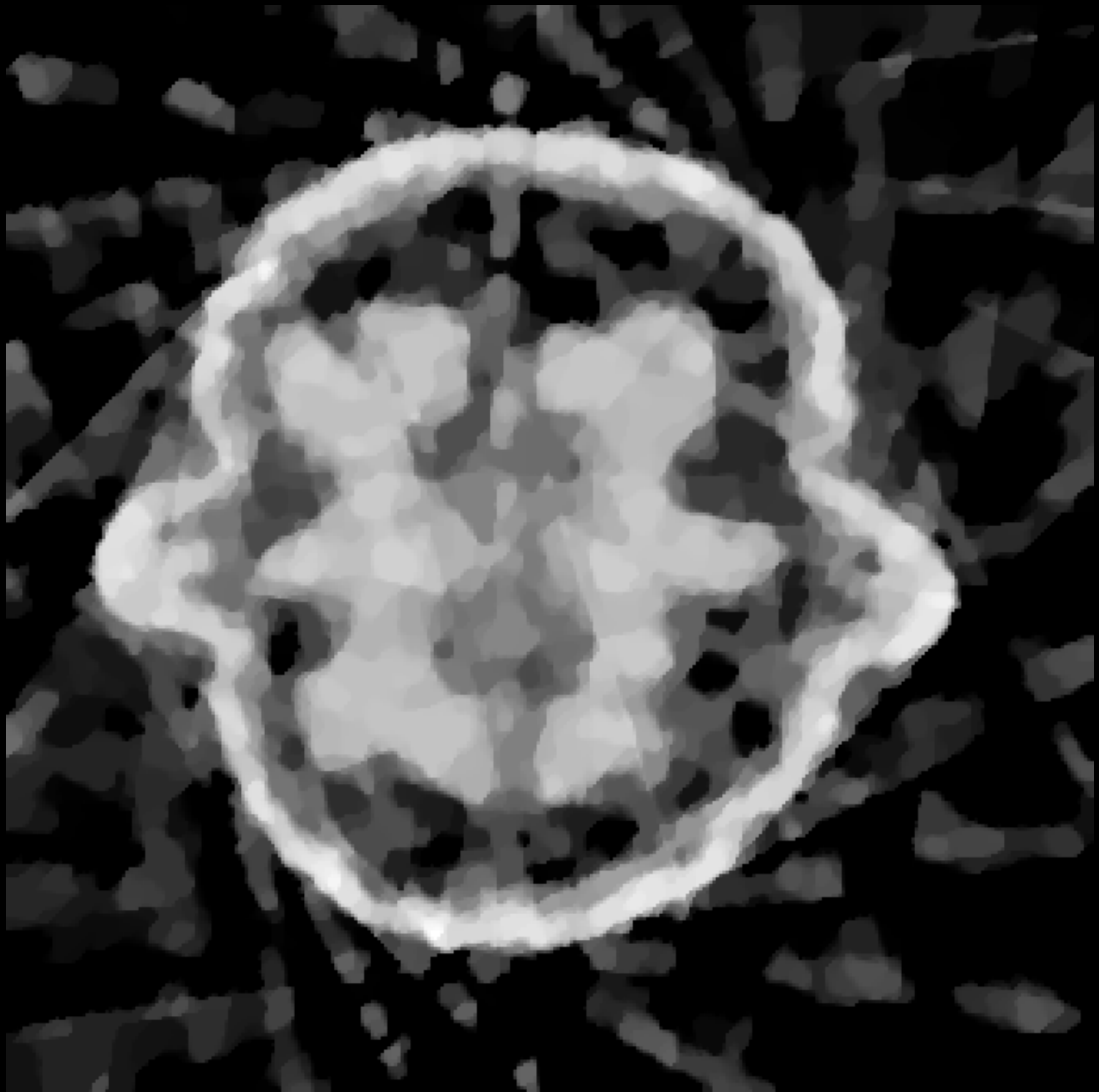
Filtered back-projection

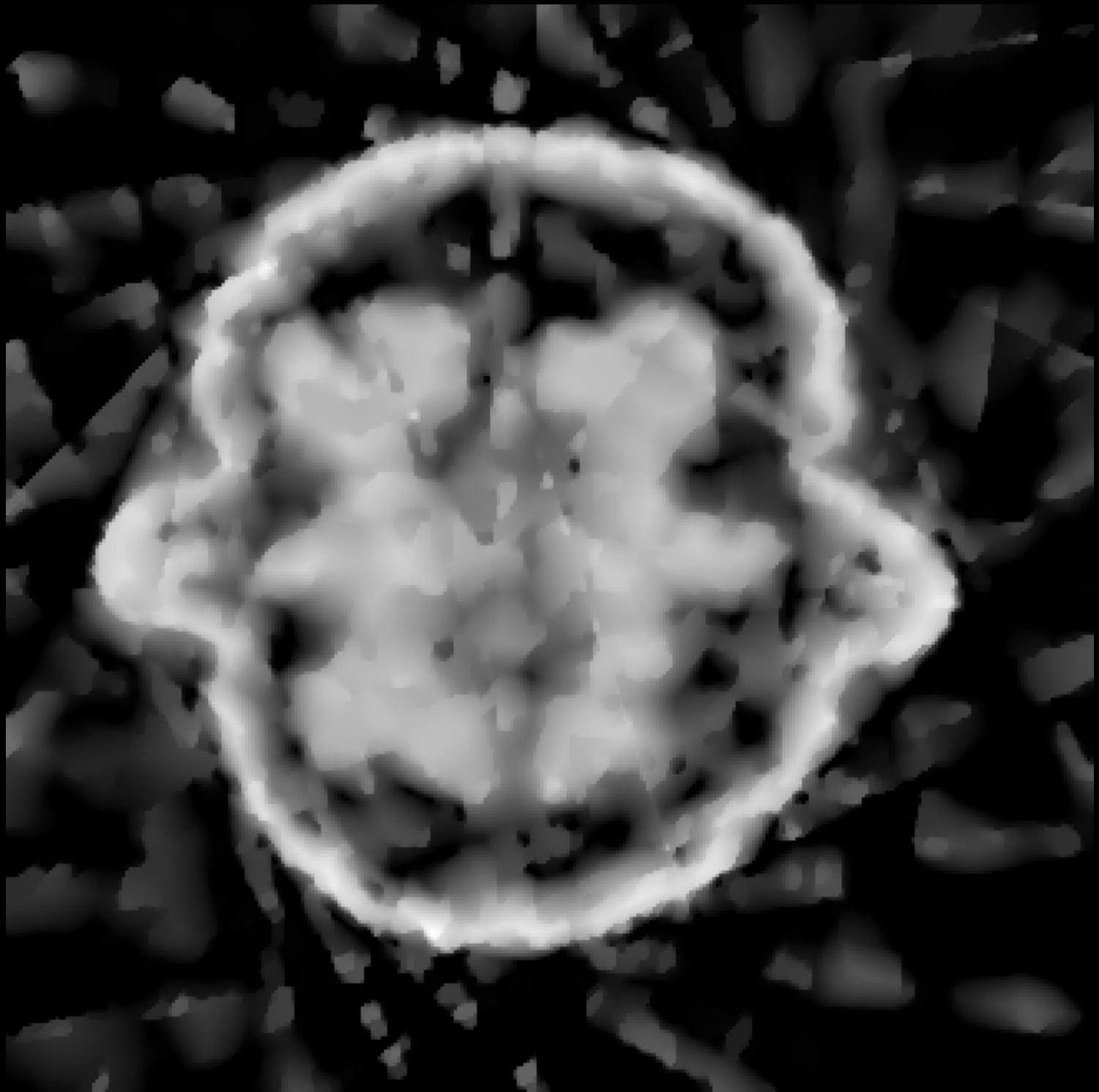


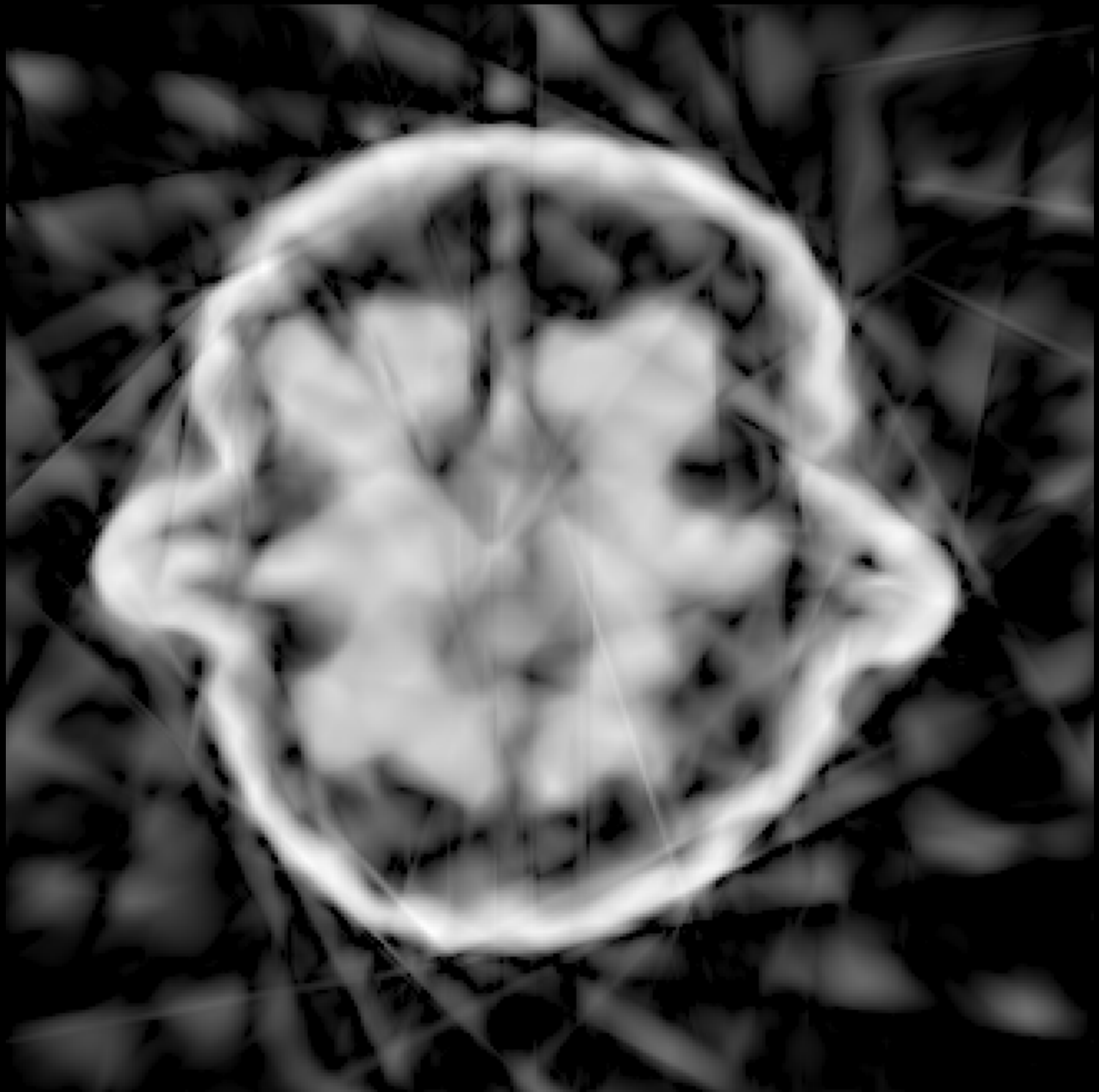
Constrained TV regularization

$$\arg\min_{f \in \mathbb{R}^n_+} \left\{ \|Af - m\|_2^2 + \alpha \|\nabla f\|_1 \right\}$$



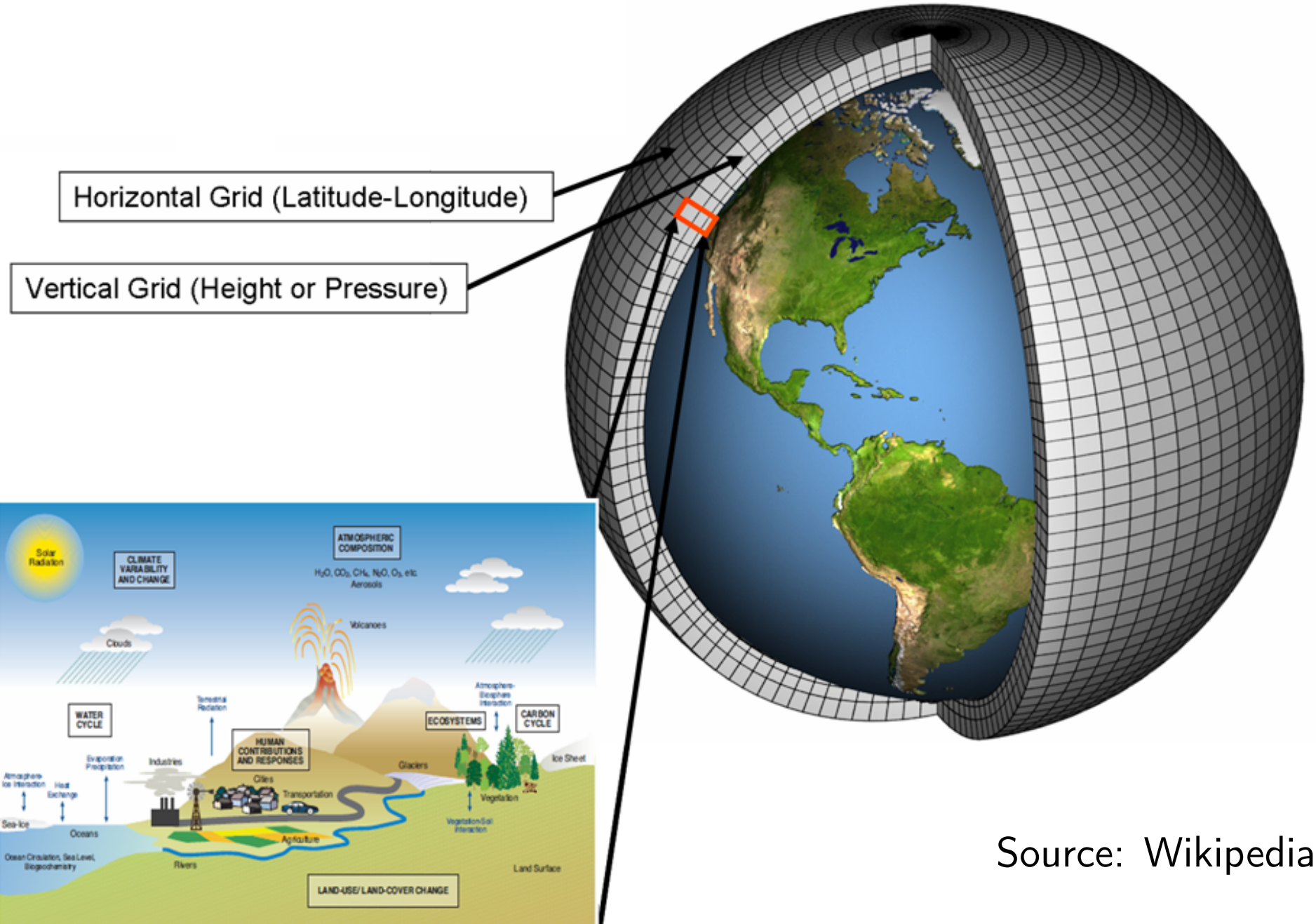






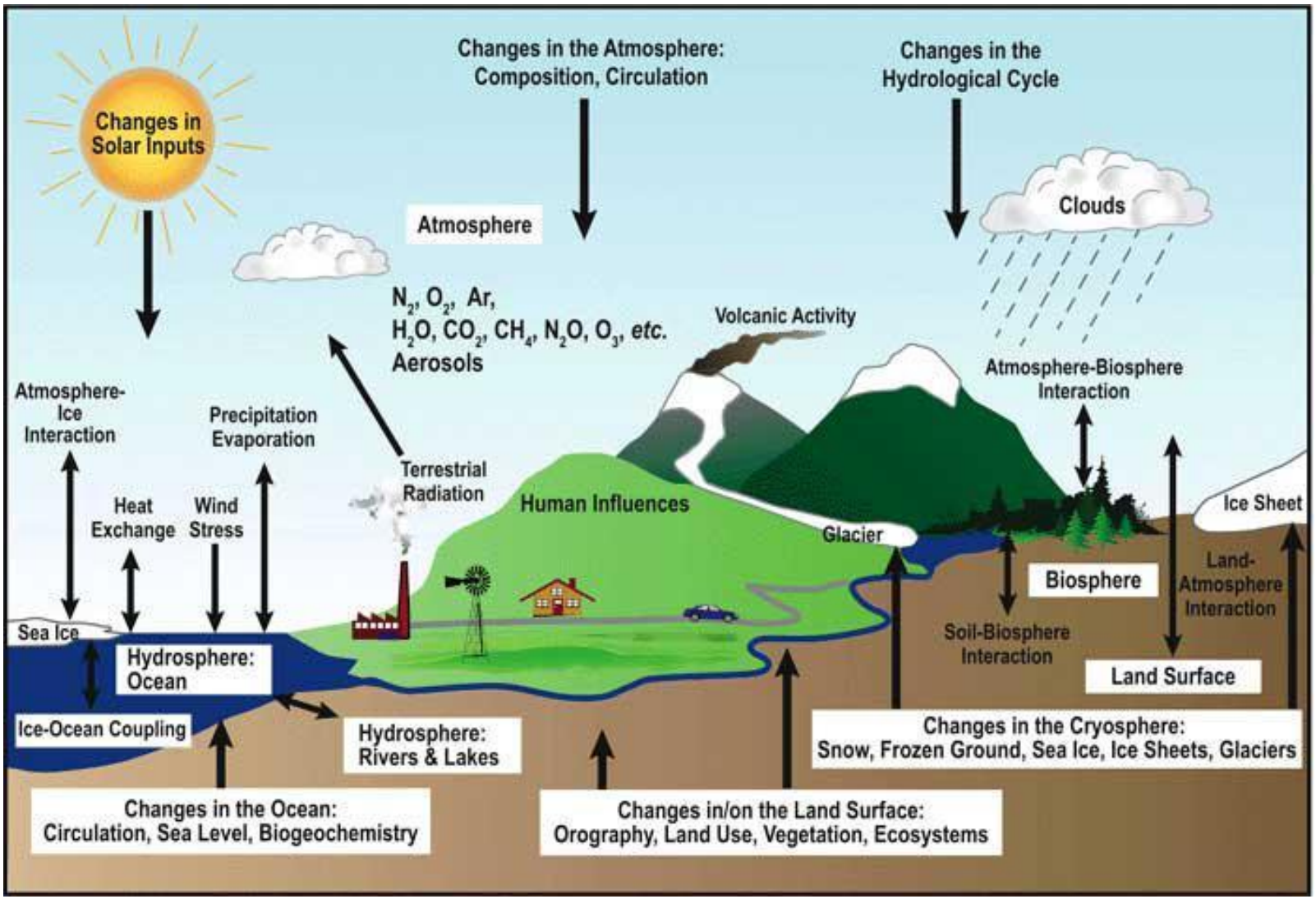
Outline

Climate change is predicted using climate models



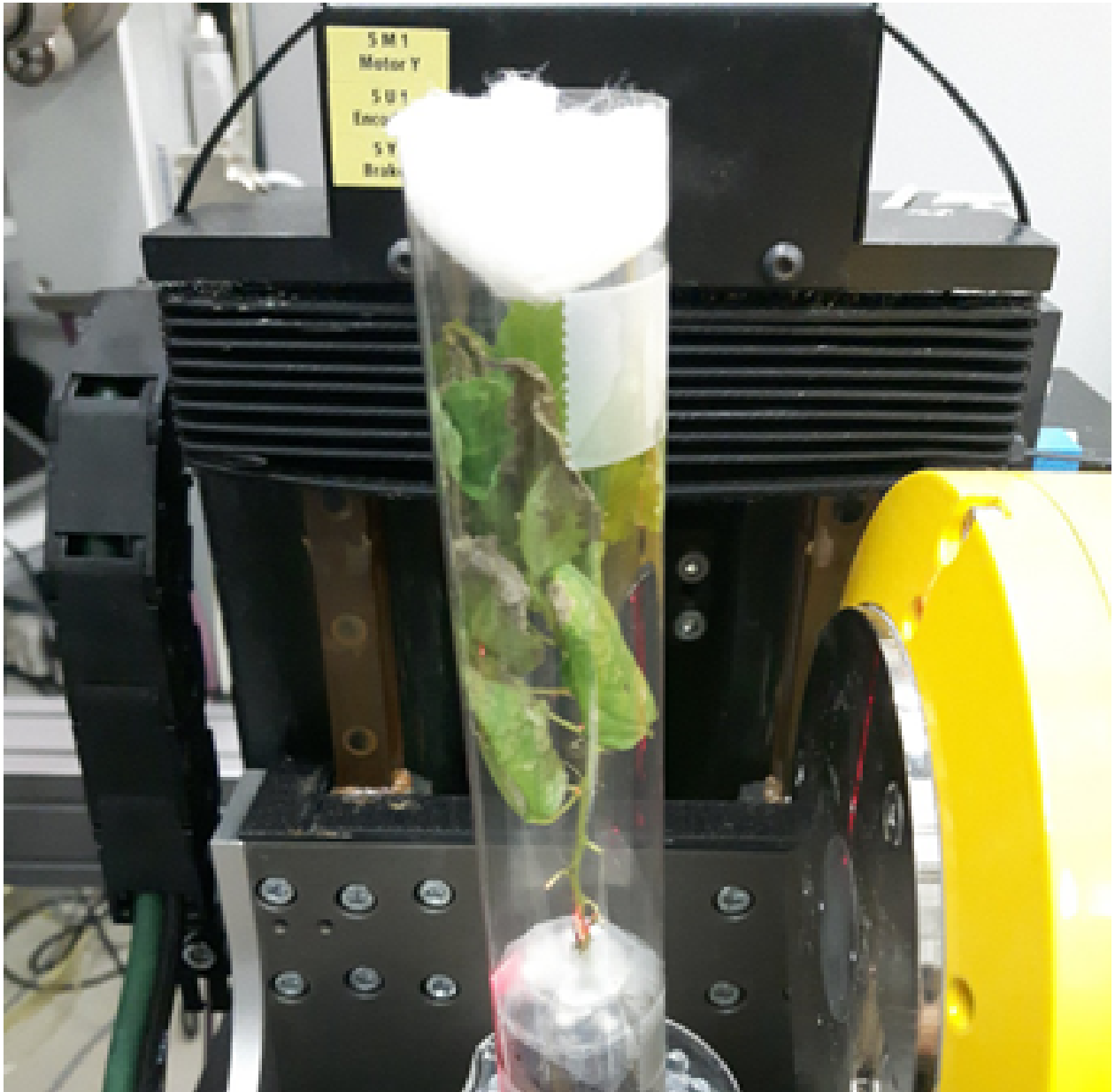
Source: Wikipedia

Climate models have a lot of details, and plant metabolism is crucial to model accurately



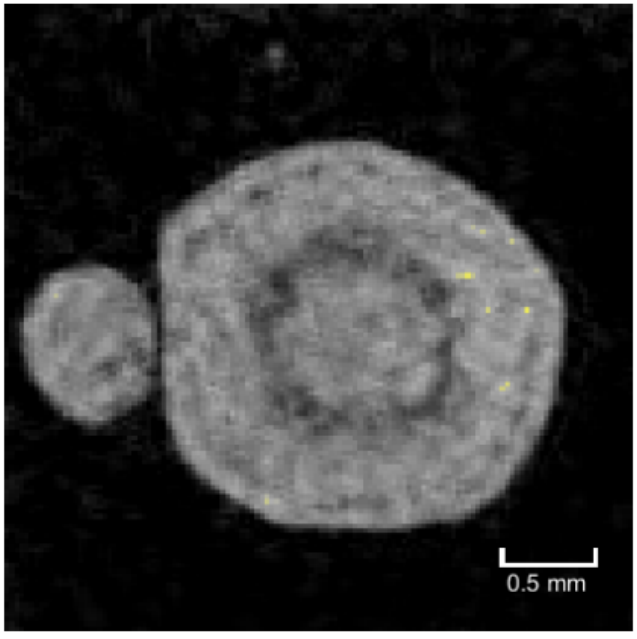
Le Treut, Somerville, Cubasch, Ding, Mauritzen, Mokssit, Peterson & Prather 2007

Tomography study jointly with physicists, biologists, radiochemists and climate scientists

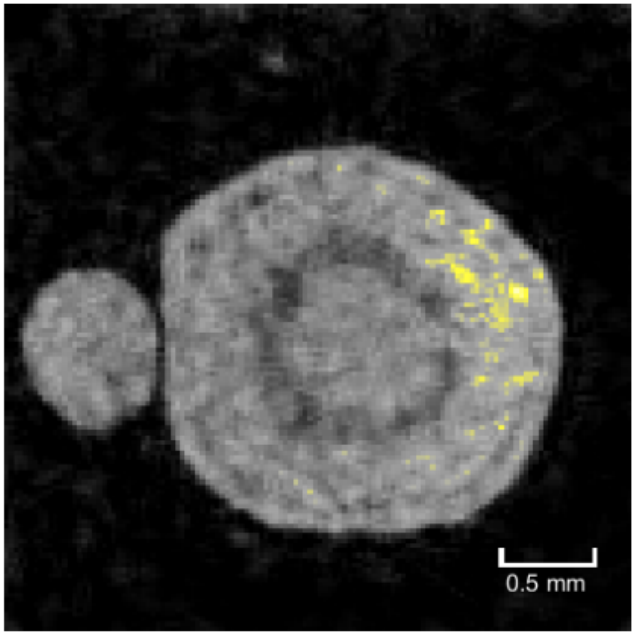


Time-dependent sparse tomography reveals the movement of iodine in the phloem

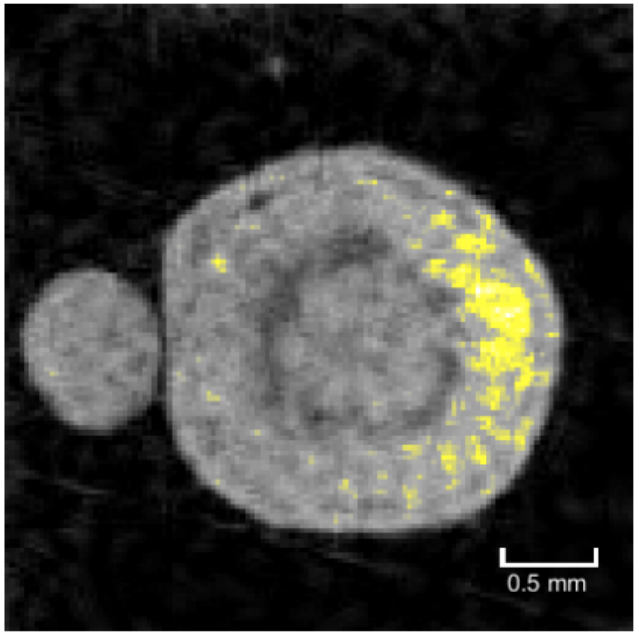
0 minutes



166 minutes



235 minutes



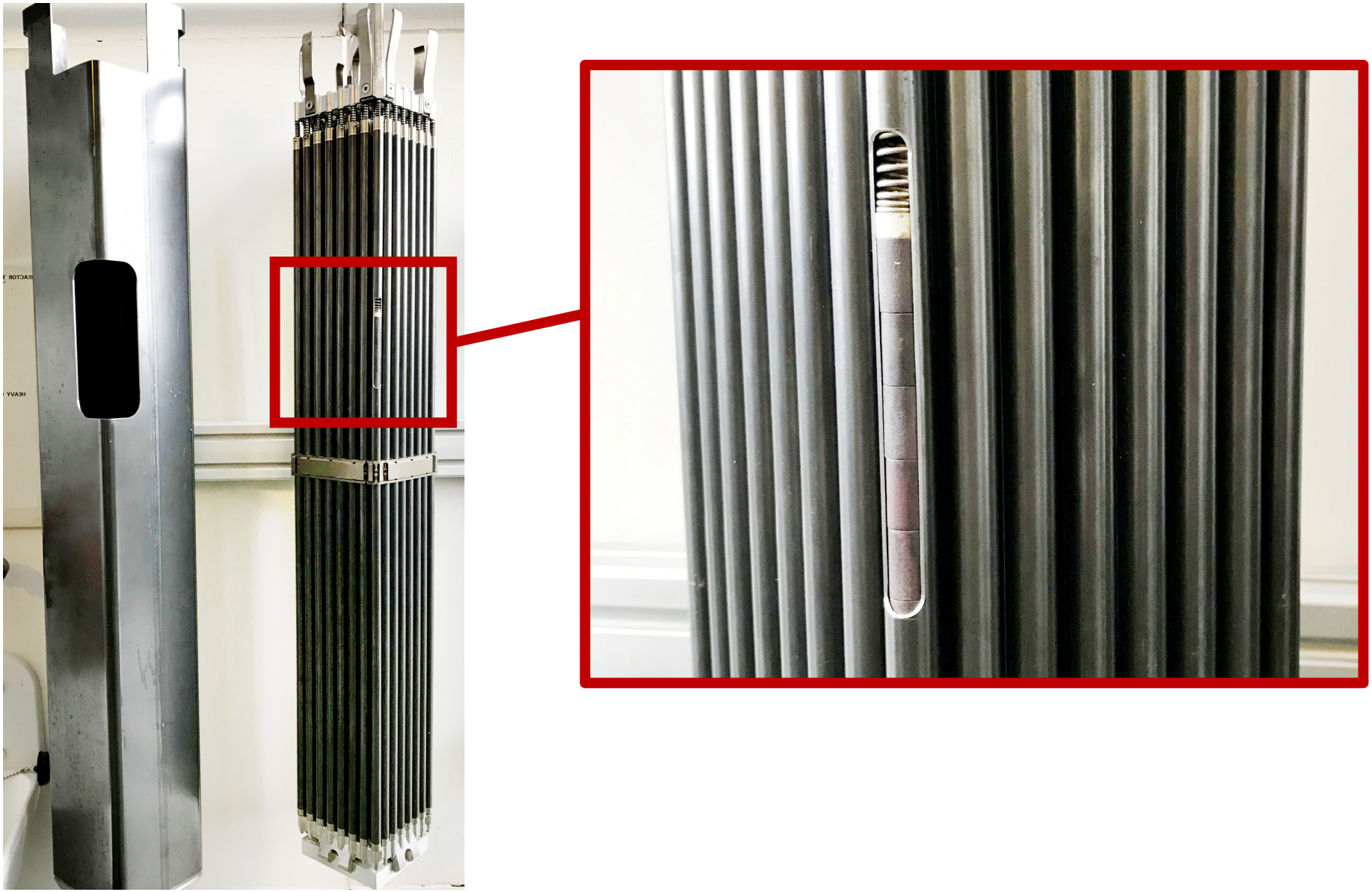
Bubba, Heikkilä, Help, Huotari, Salmon & S, in press

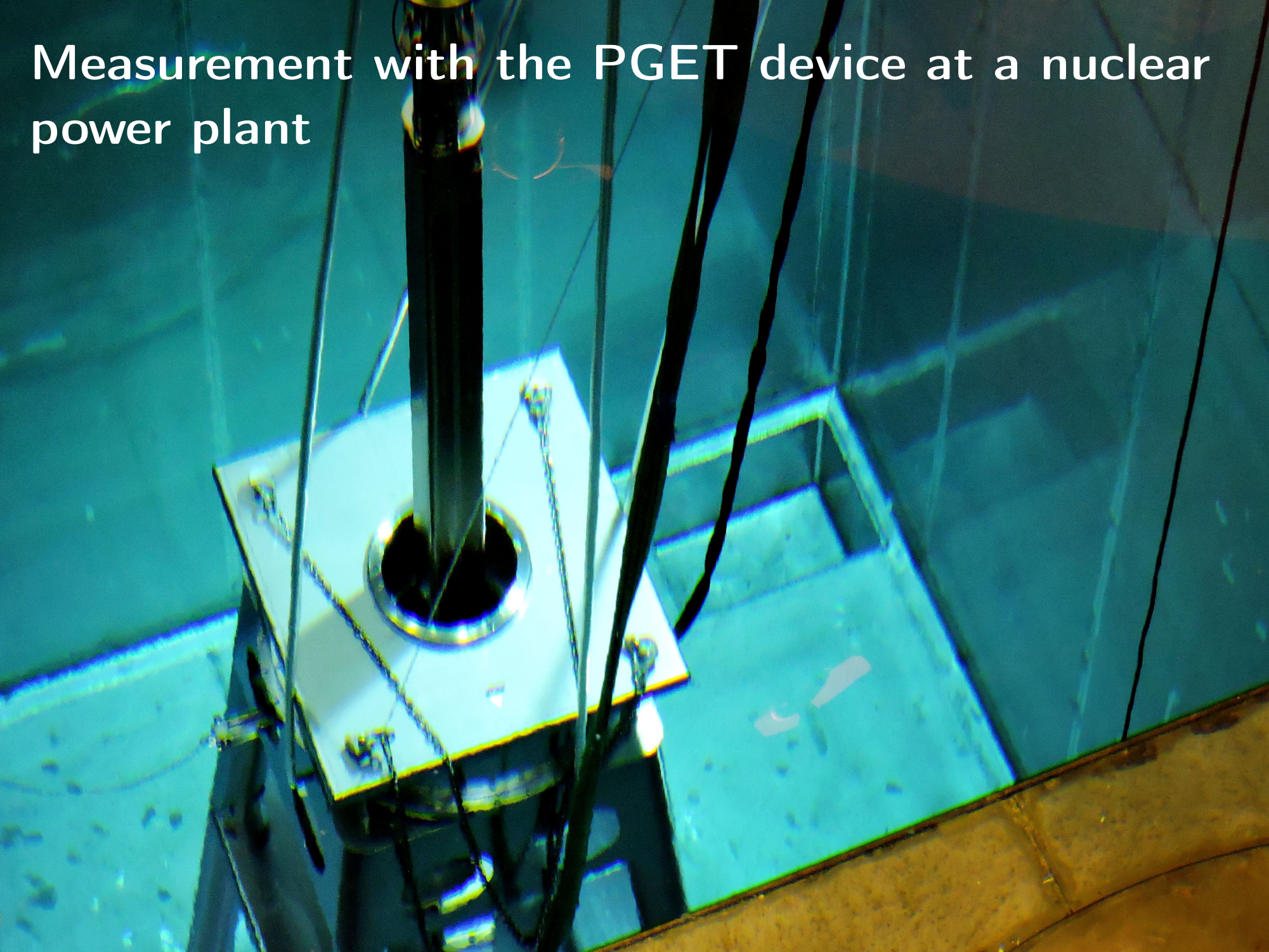
Carbon emissions of energy production (median)

Method	CO_2-gram/kWh
Coal	820
Gas	490
Biomass	230
Solar	41
Geothermal	38
Hydropower	24
Nuclear	12
Wind	11

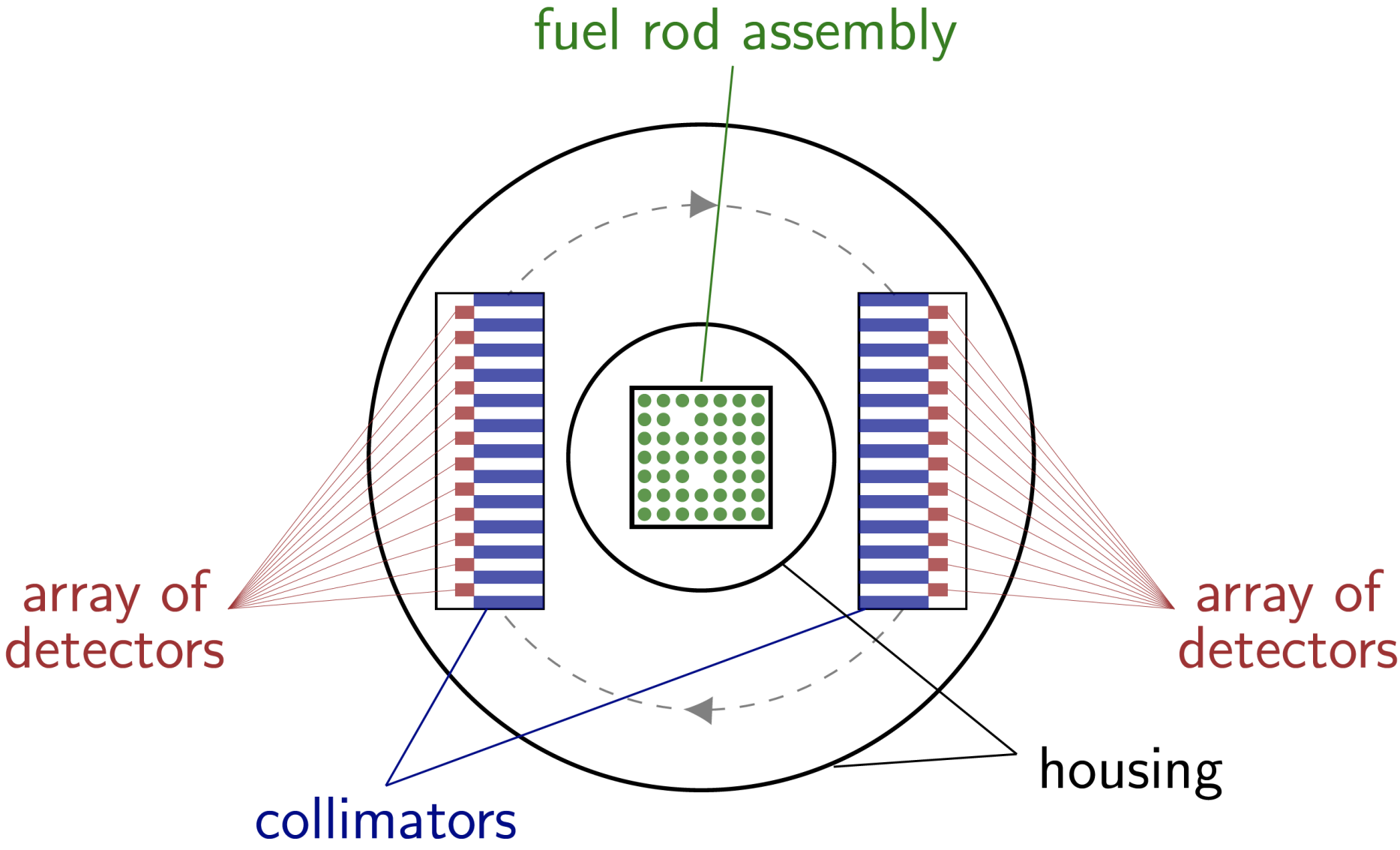
Source: IPCC; see page 7 in the document
https://www.ipcc.ch/site/assets/uploads/2018/02/ipcc_wg3_ar5_annex-iii.pdf

A nuclear fuel assembly consists of rods filled with pellets containing uranium



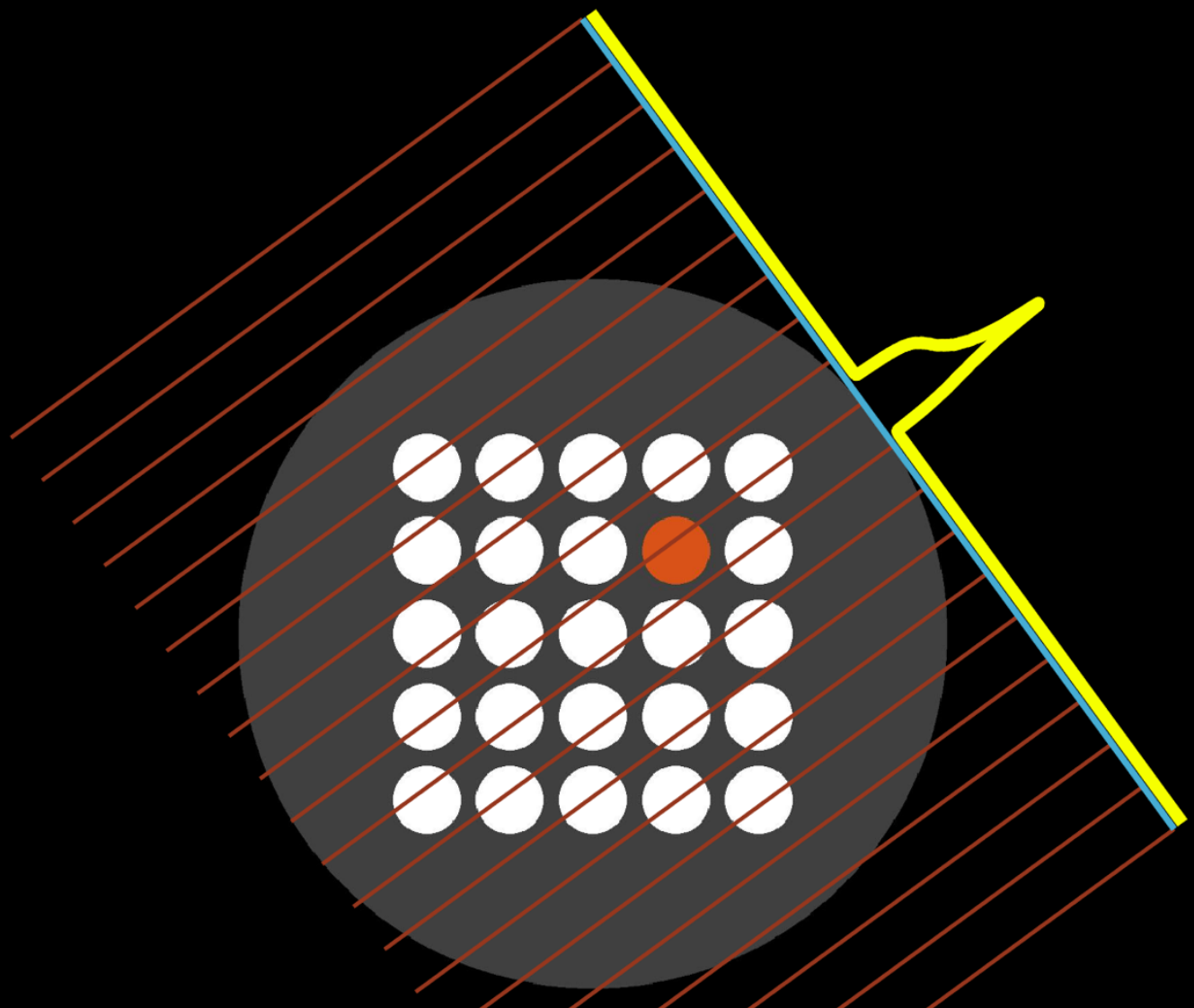

Measurement with the PGET device at a nuclear power plant

Measurement geometry of the PGET device

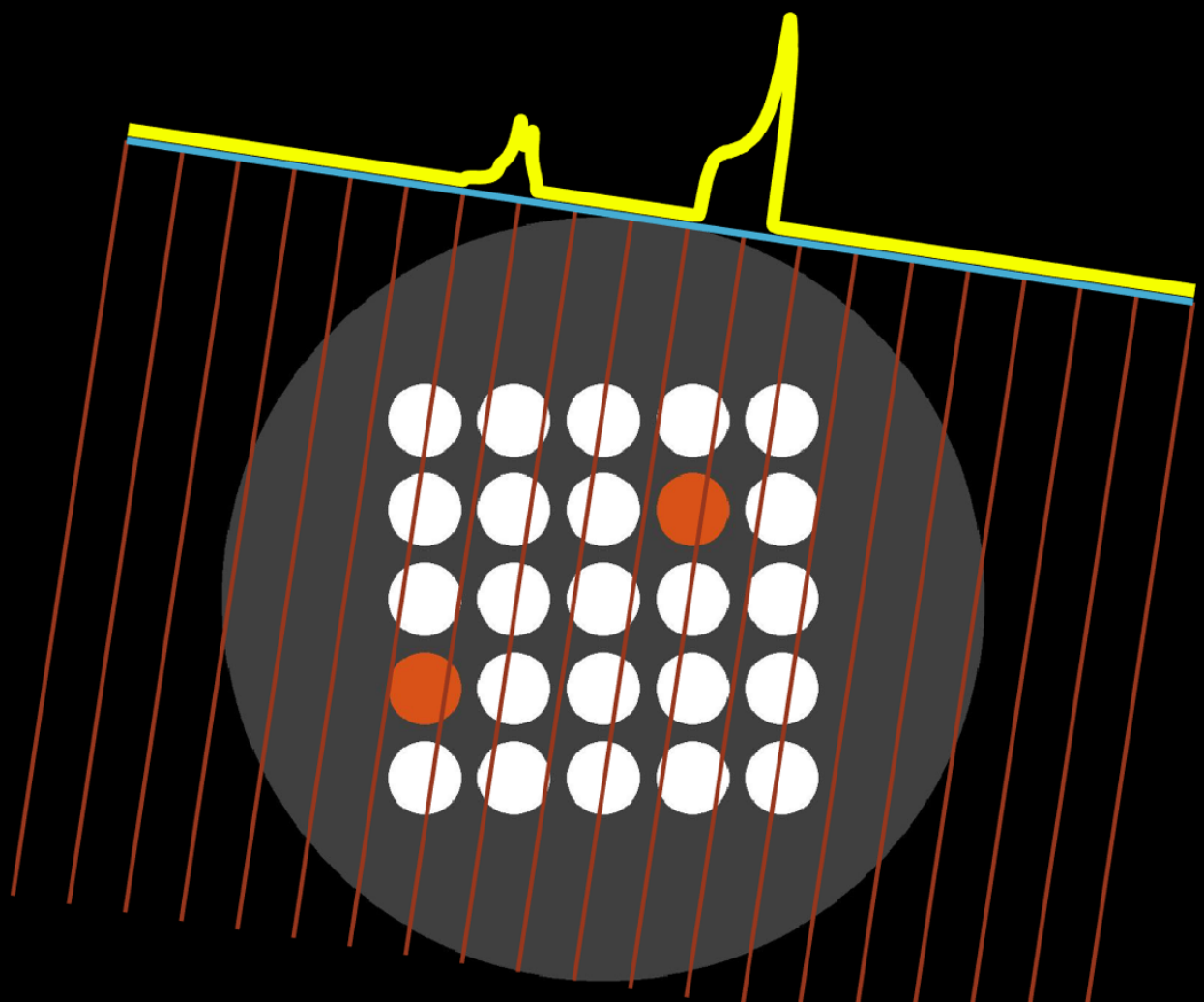


This is how the PGET device collects data

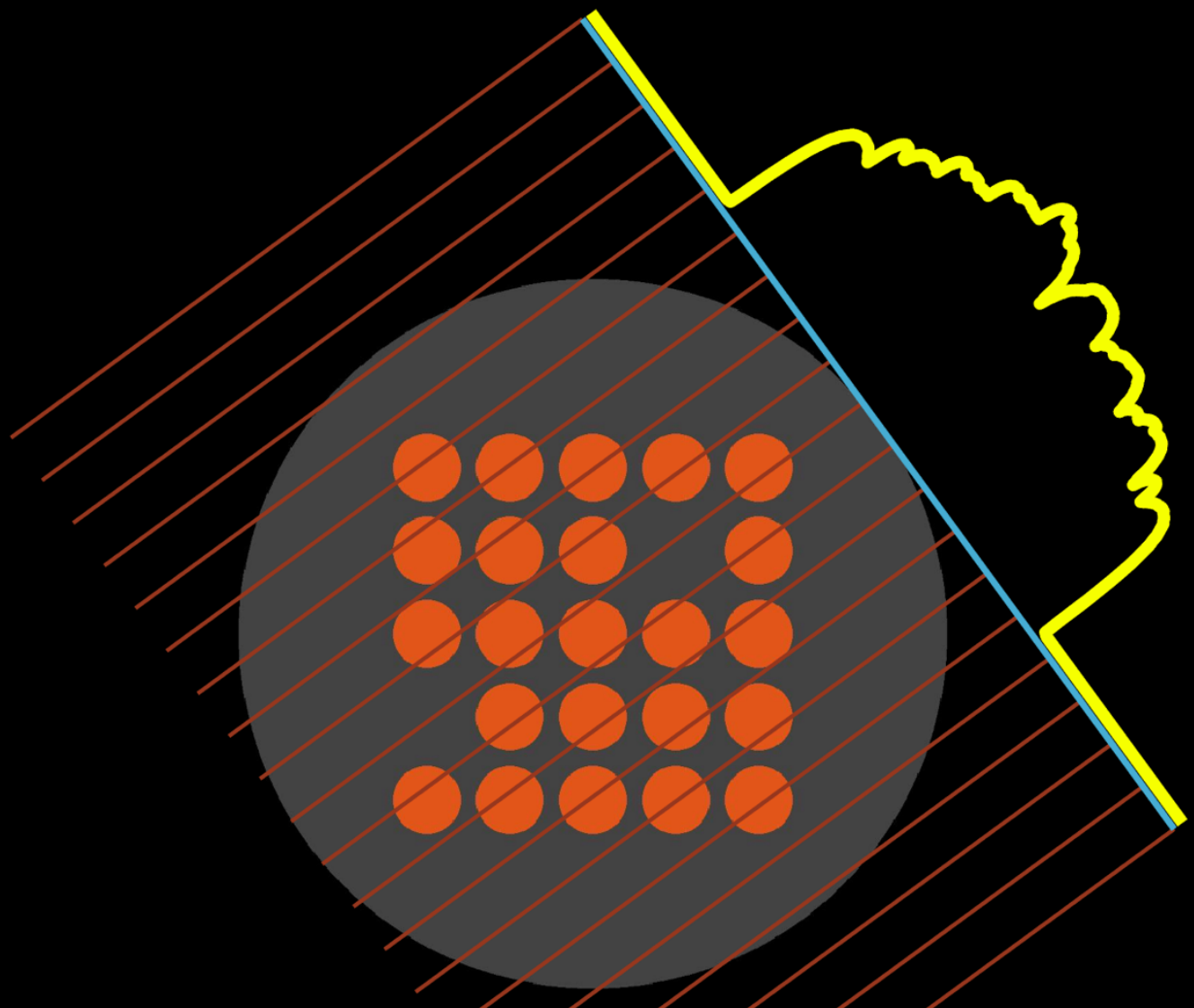
Data is collected by rotating the system around the fuel assembly



Data is collected by rotating the system around the fuel assembly

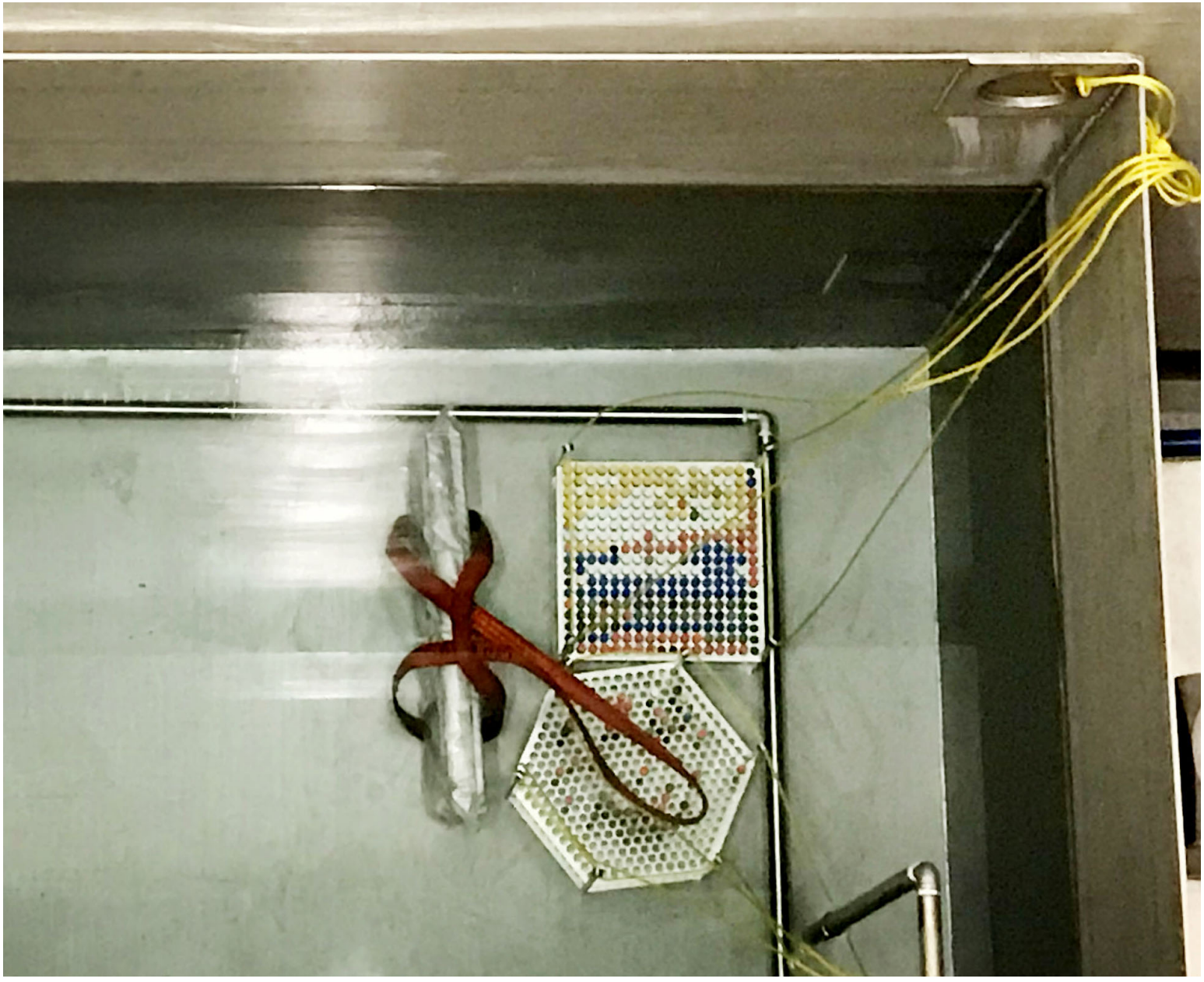


Data is collected by rotating the system around the fuel assembly

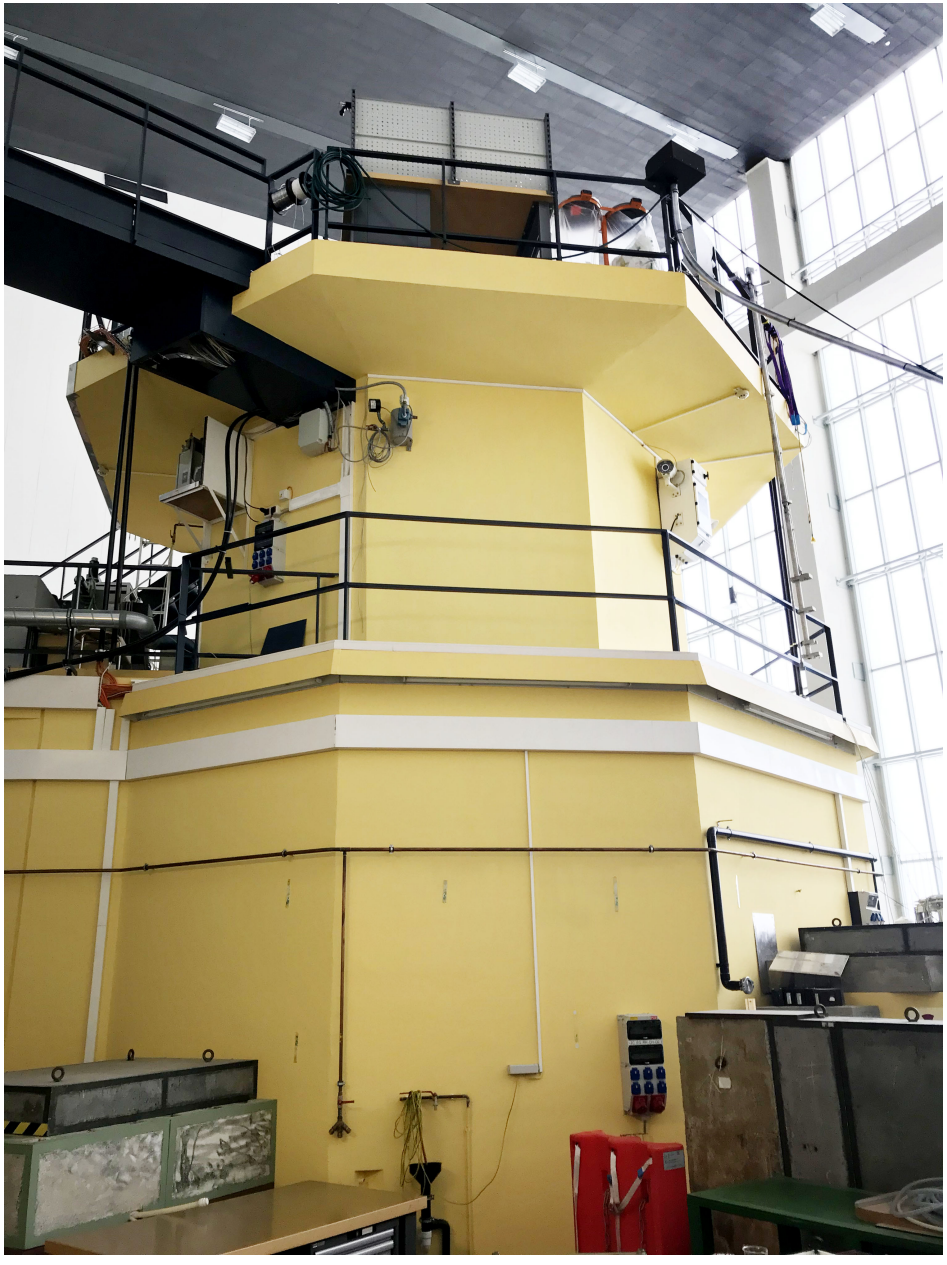


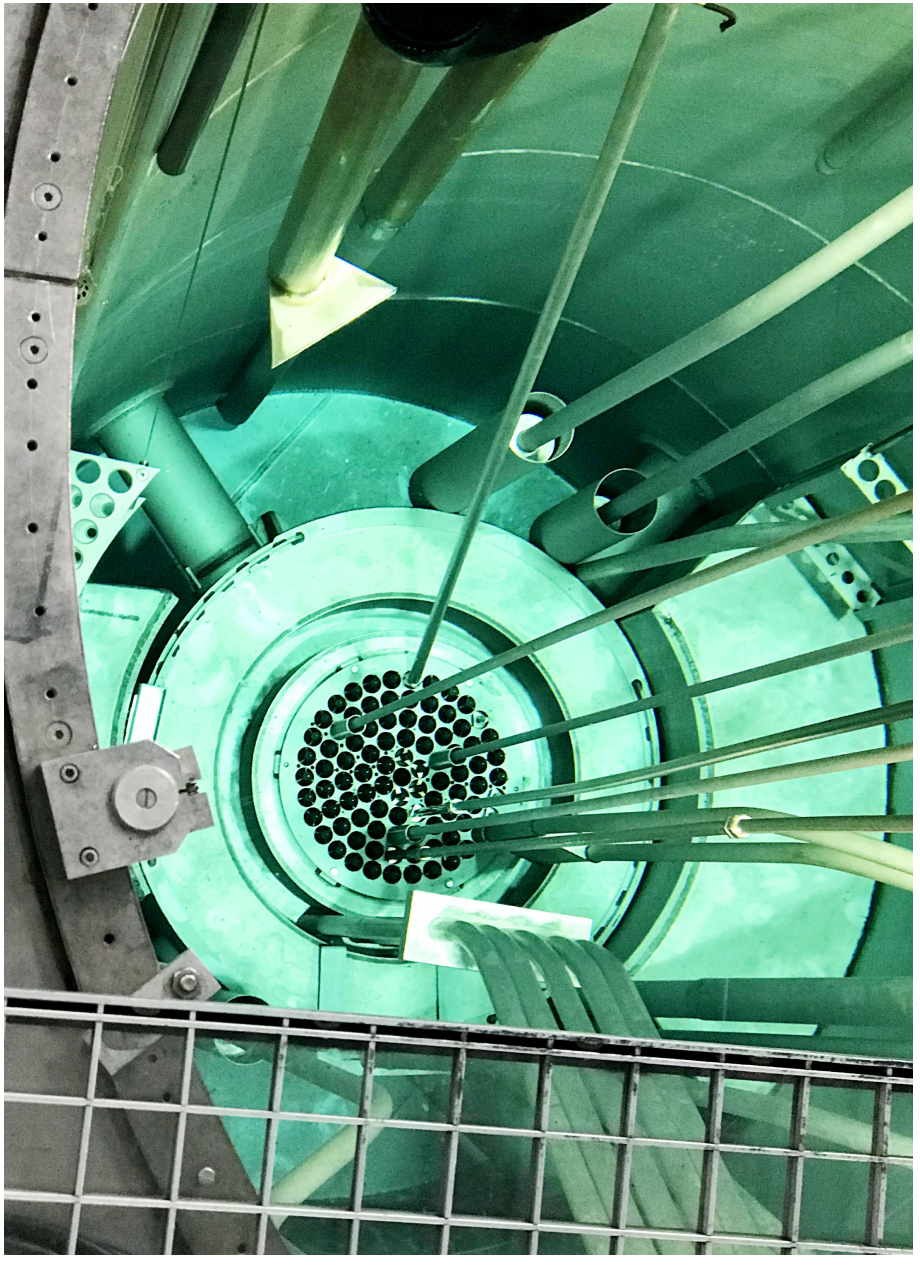
How We Won Silver in IAEA PGET Challenge

These are the mock-up fuel assemblies



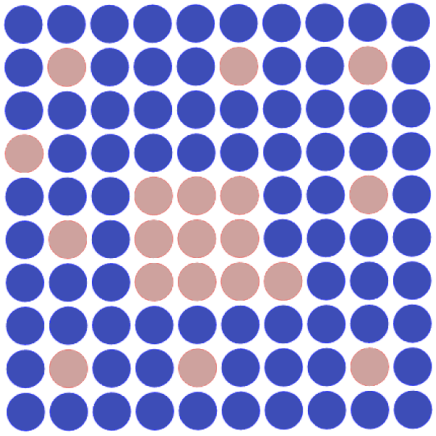
The cobolt “fuel rods” were activated in a reactor



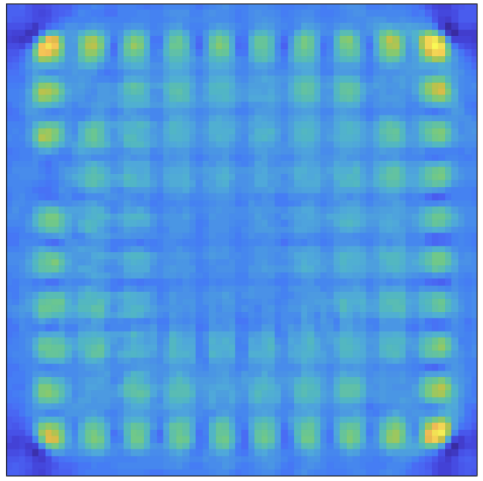


Reconstruction by Filtered Back-Projection

Ground truth,
present / missing



Activity
reconstruction

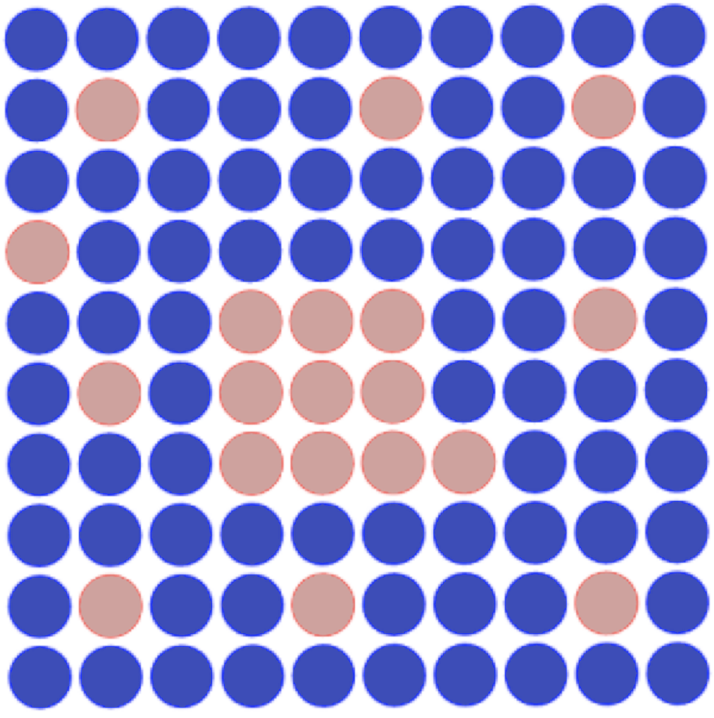


Attenuation
reconstruction

Not applicable

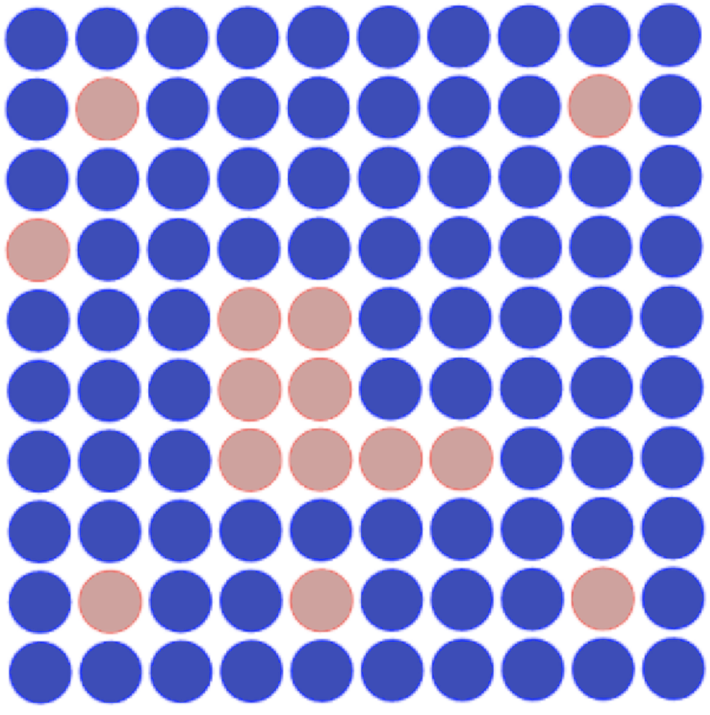
Classification by Filtered Back-Projection

Ground truth



present/missing

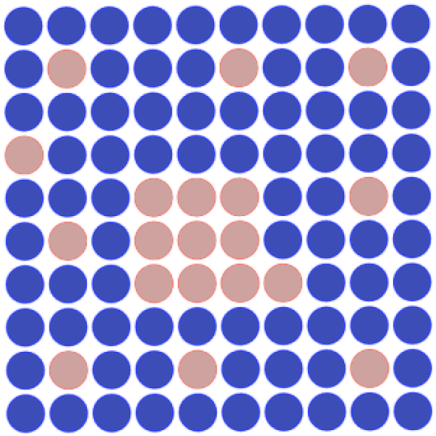
Classification



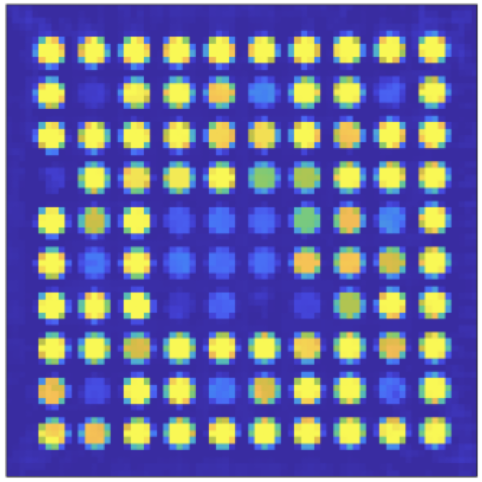
present/missing

Reconstruction by geometry-aware prior

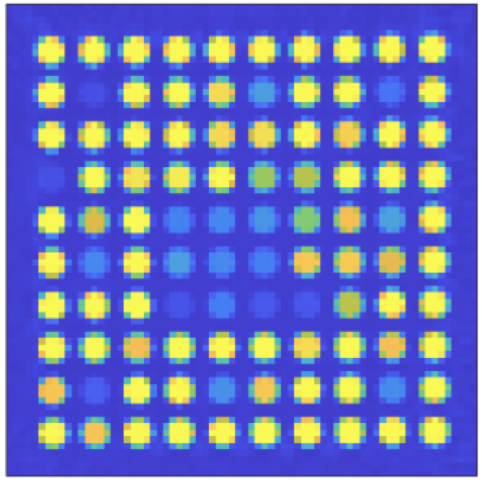
Ground truth,
present / missing



Activity
reconstruction



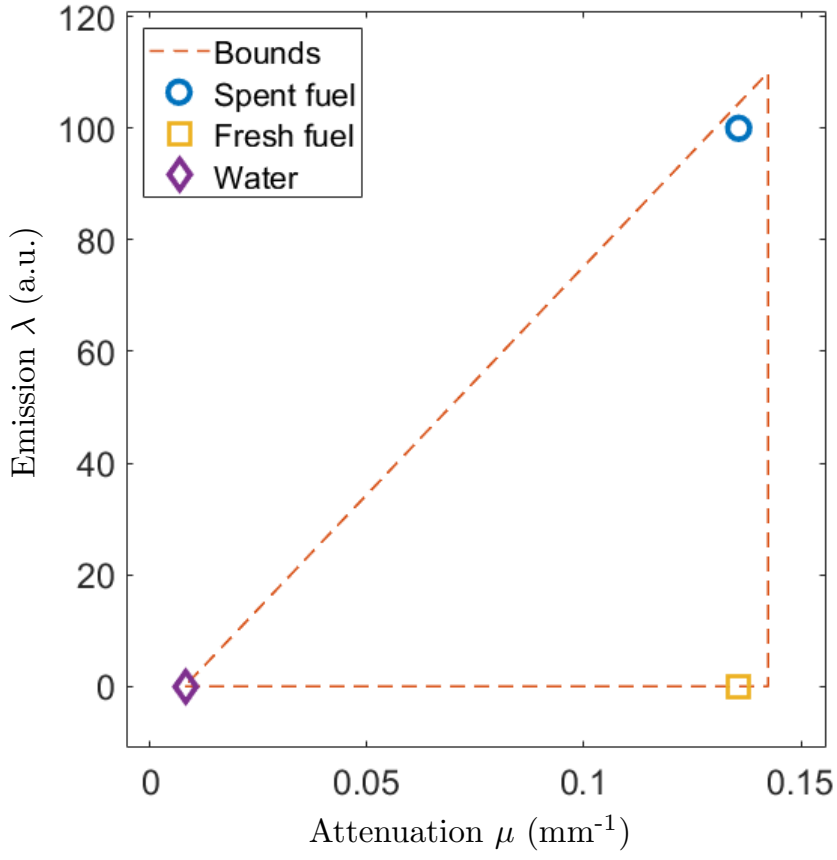
Attenuation
reconstruction



One piece of *a priori* information we put into the reconstruction is the physicality of materials

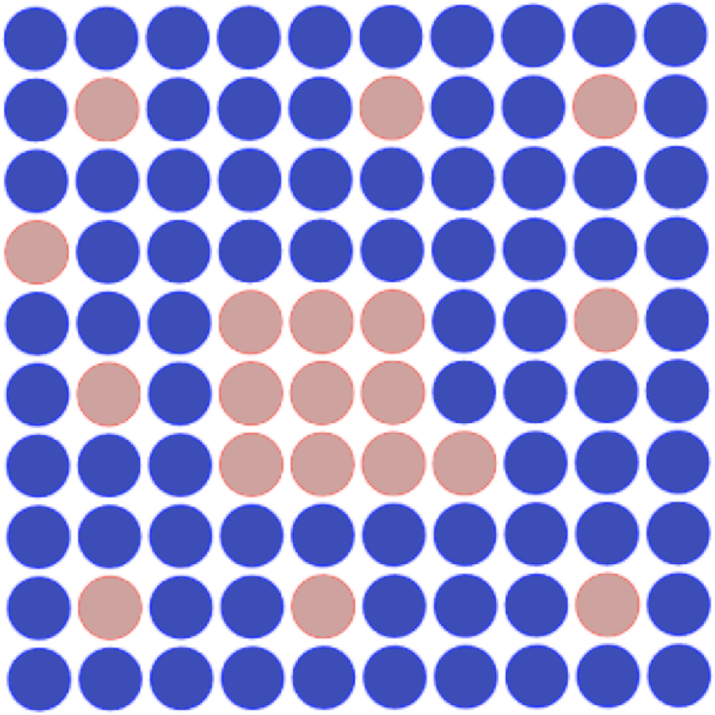
Need to set bounds for the emission and attenuation values in the minimization problem to produce reasonable images.

- Excludes the possibility of a material with high emission but low attenuation value.
- Some way of estimating these bounds before the minimization is needed.



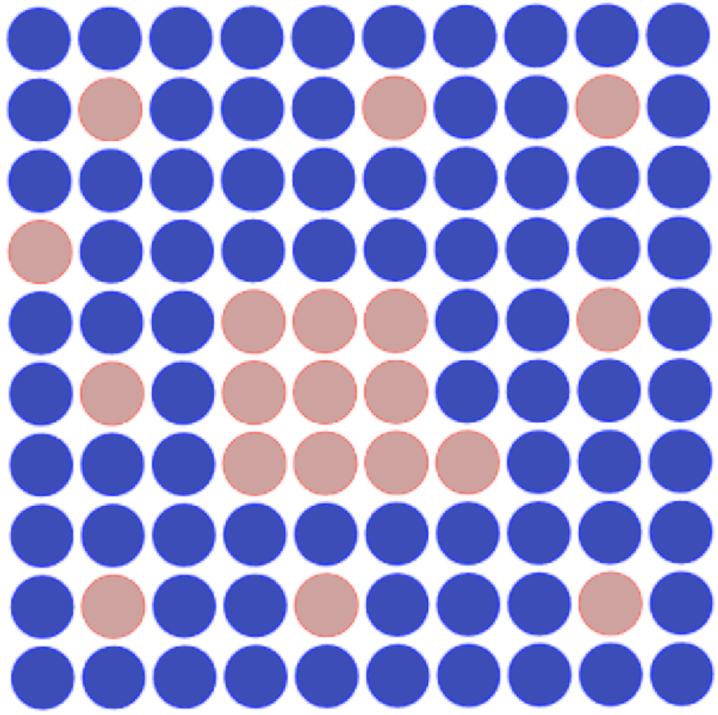
Classification by geometry-aware prior

Ground truth



present/missing

Classification



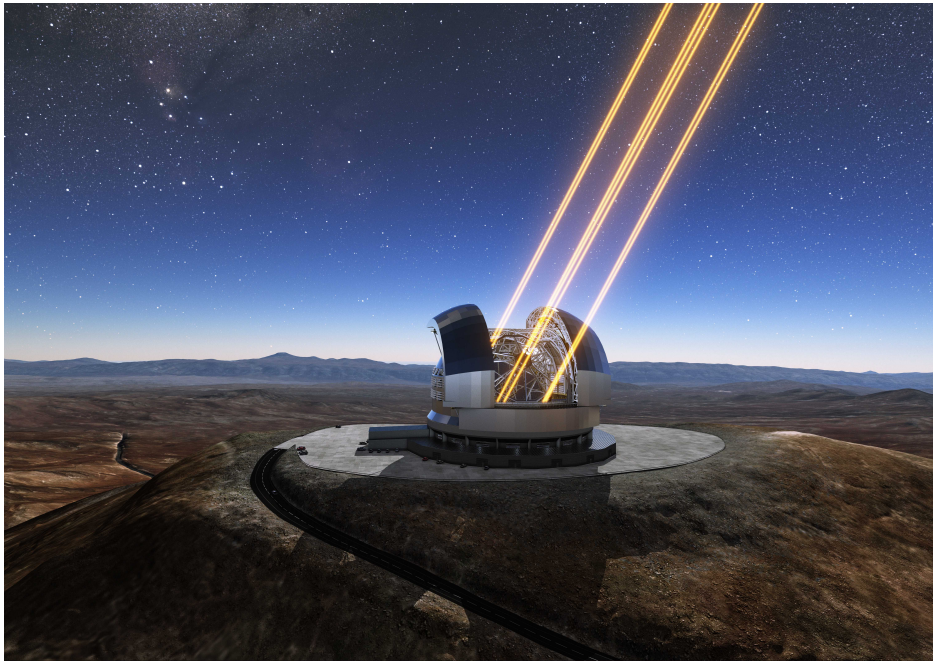
present/missing

[Backholm, Bubba, Bélanger-Champagne, Helin, Dendooven & S 2020]

Outline

Tomography appears in adaptive optics

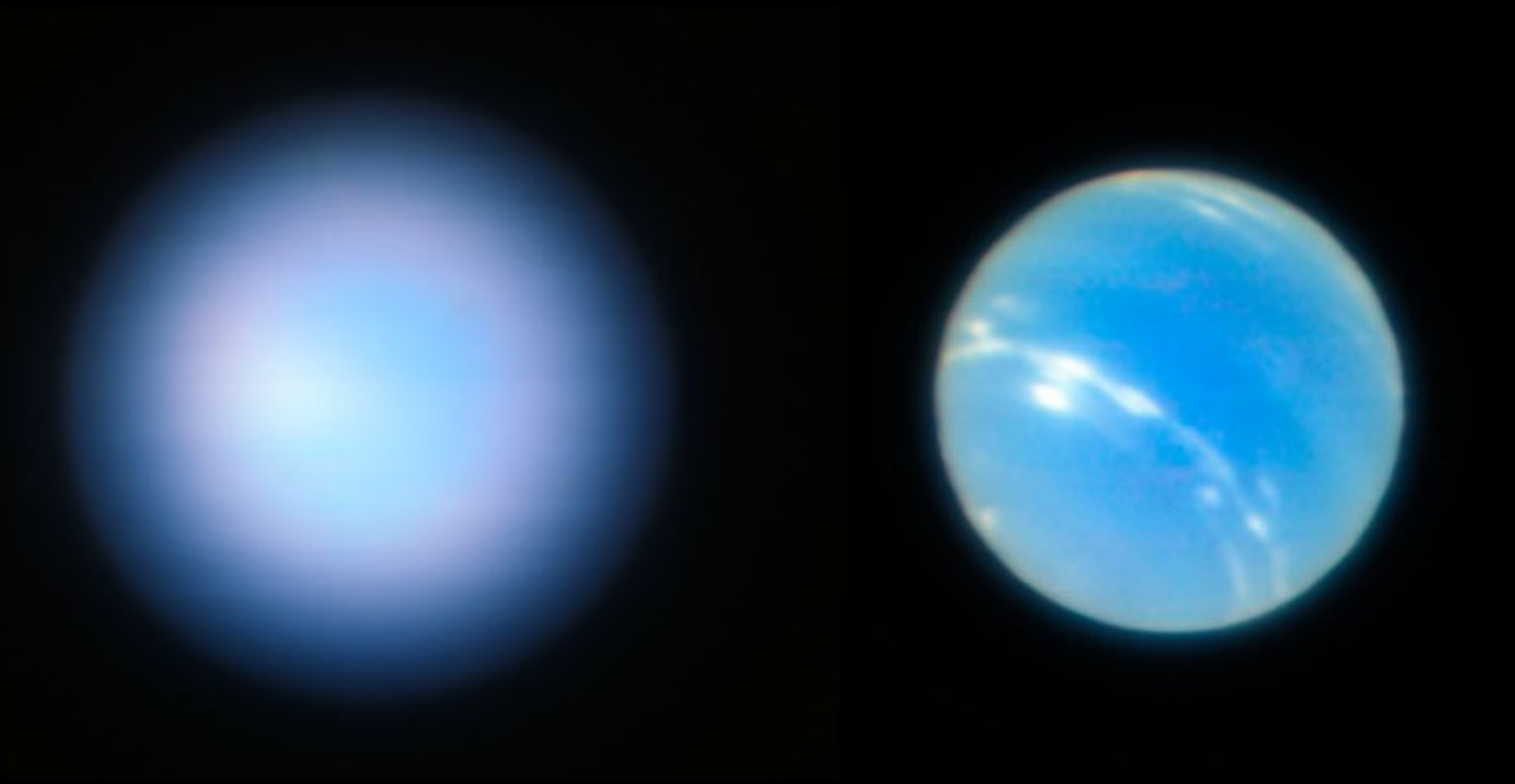
- Modern telescope imaging suffers from turbulence in the atmosphere
  $\Rightarrow$ blurring of images
- Adaptive optics corrects the perturbed incoming light in real-time
- Major challenge in wide-field AO: atmospheric tomography



European Extremely Large Telescope (2024)

Helin, Kindermann, Lehtonen & Ramlau 2018
Yudytskiy, Helin & Ramlau 2014

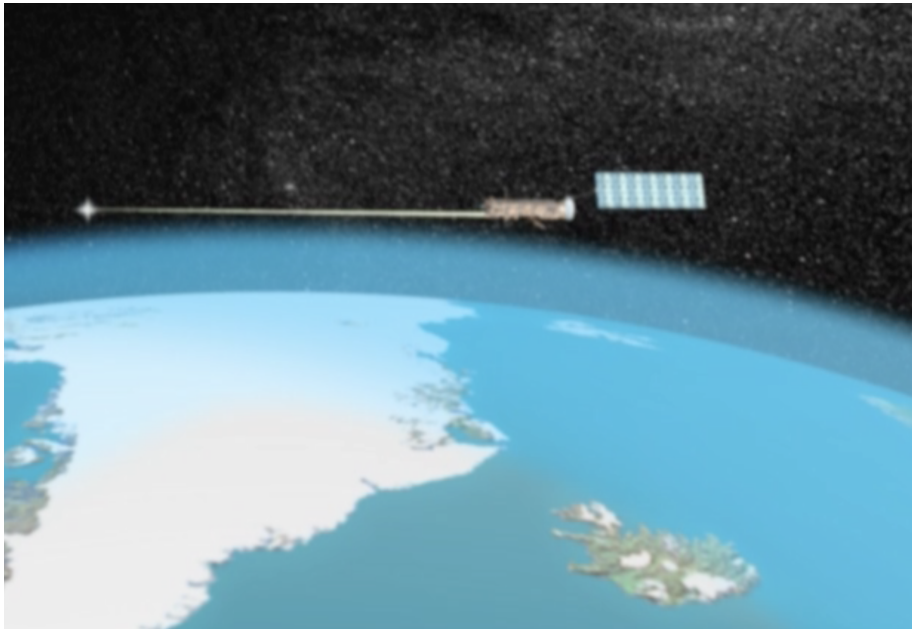
Photograph of planet Neptune with and without adaptive optics (image: ESO/P. Weilbacher)

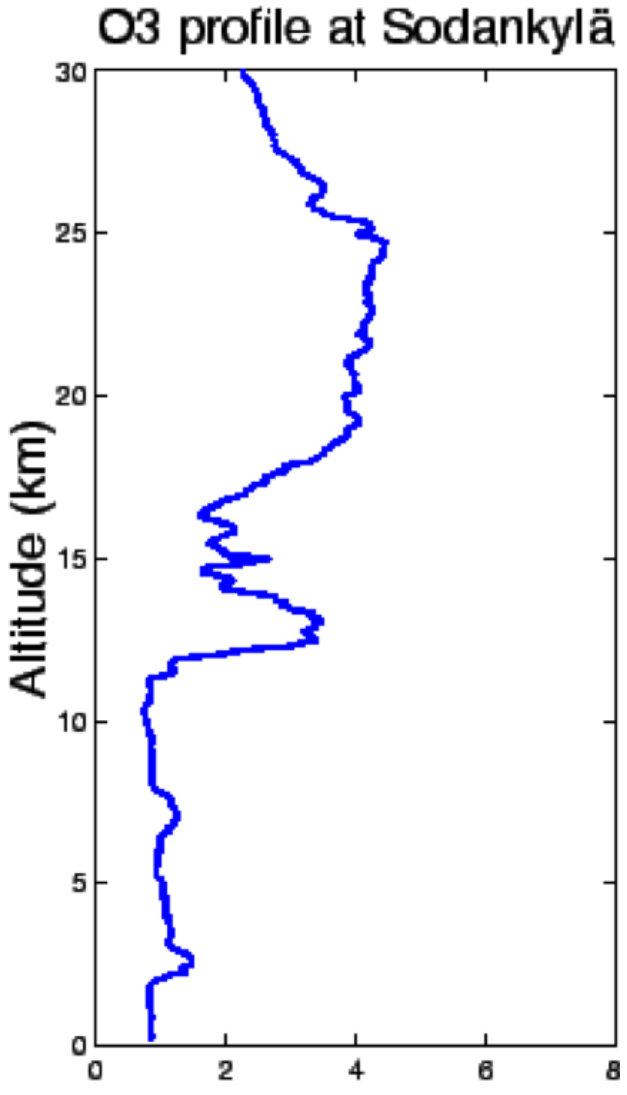


No adaptive optics

Adaptive optics

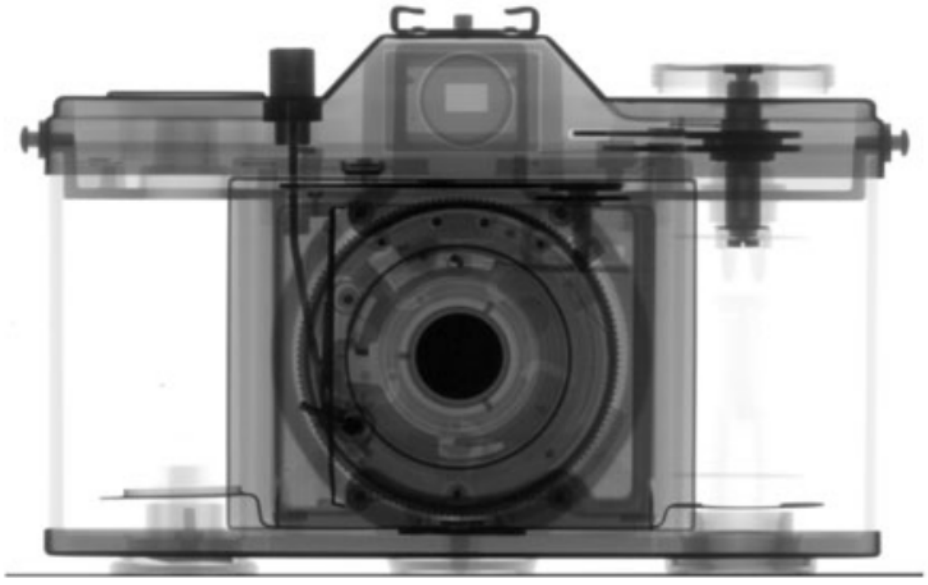
The mathematics of X-ray tomography can be used for recovering the ozone layer



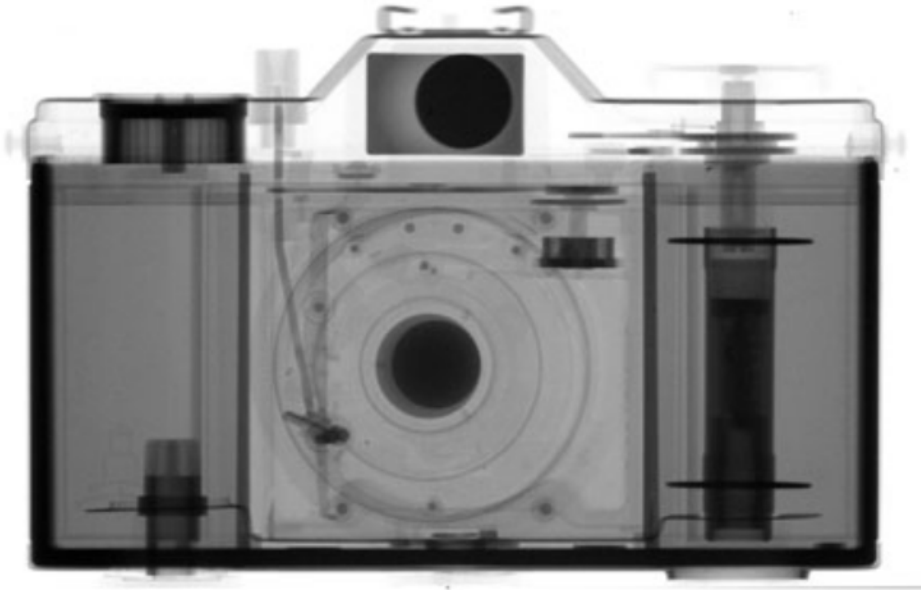


European Space Agency
Finnish Meteorological Institute
Envisat and GOMOS projects
Thanks to **Johanna Tamminen!**

Neutron beams and X-rays attenuate differently



X-ray transmission image.
Metal attenuates strongly and plastic parts are transparent.

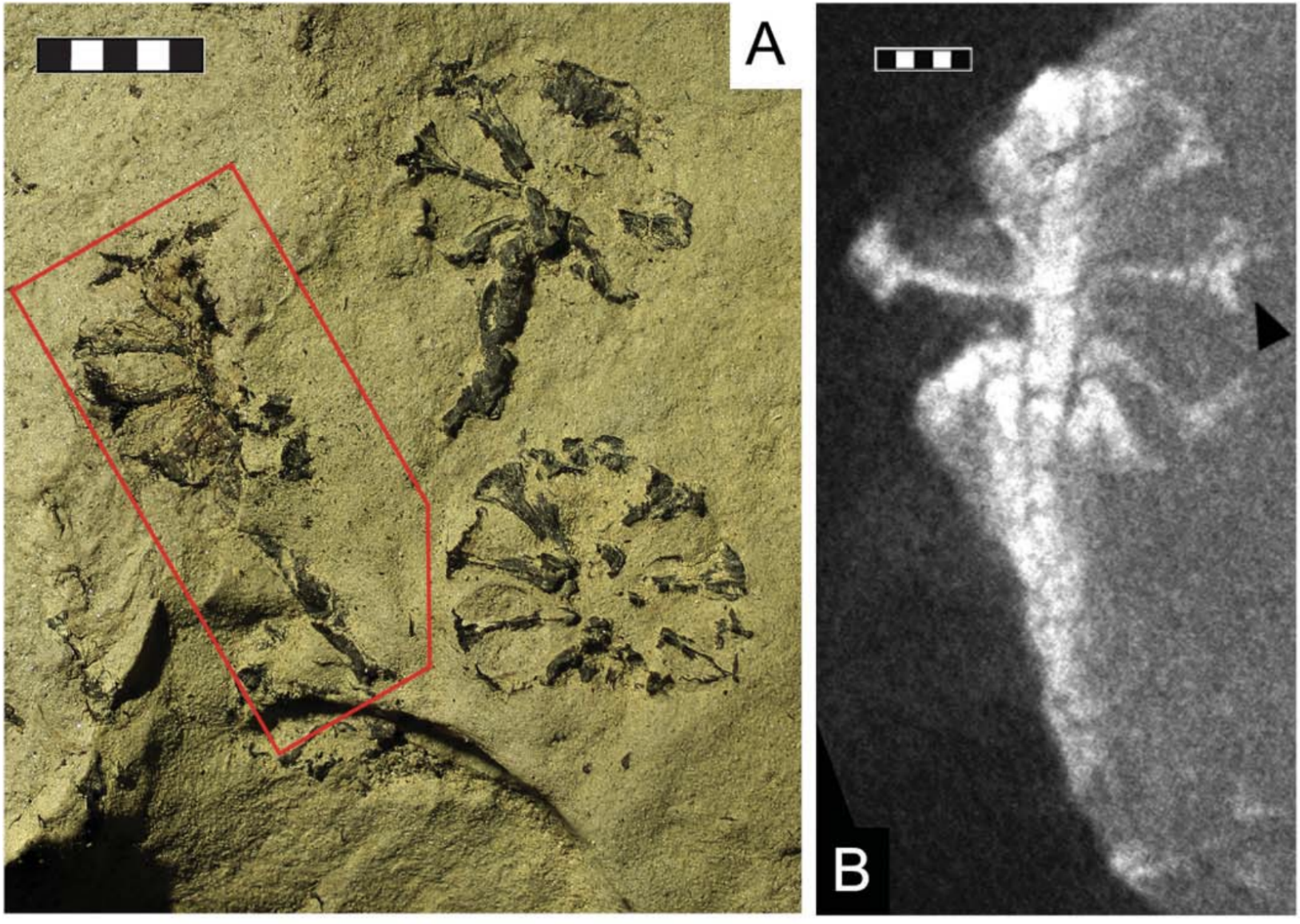


Neutron transmission image.
Metal is transparent, but plastic attenuates strongly.

Imaging with neutrons opens up new possibilities as water attenuates but metal is transparent

Video:
Anders Kaestner
Neutron Imaging and
Activation Group,
Paul Scherrer Institute

Neutron tomography of a fossilized seed cone



Mays, Cantrill, Stilwell & Bevitt 2018

Electron microscopy makes it possible to see viruses smaller than the wavelength of light

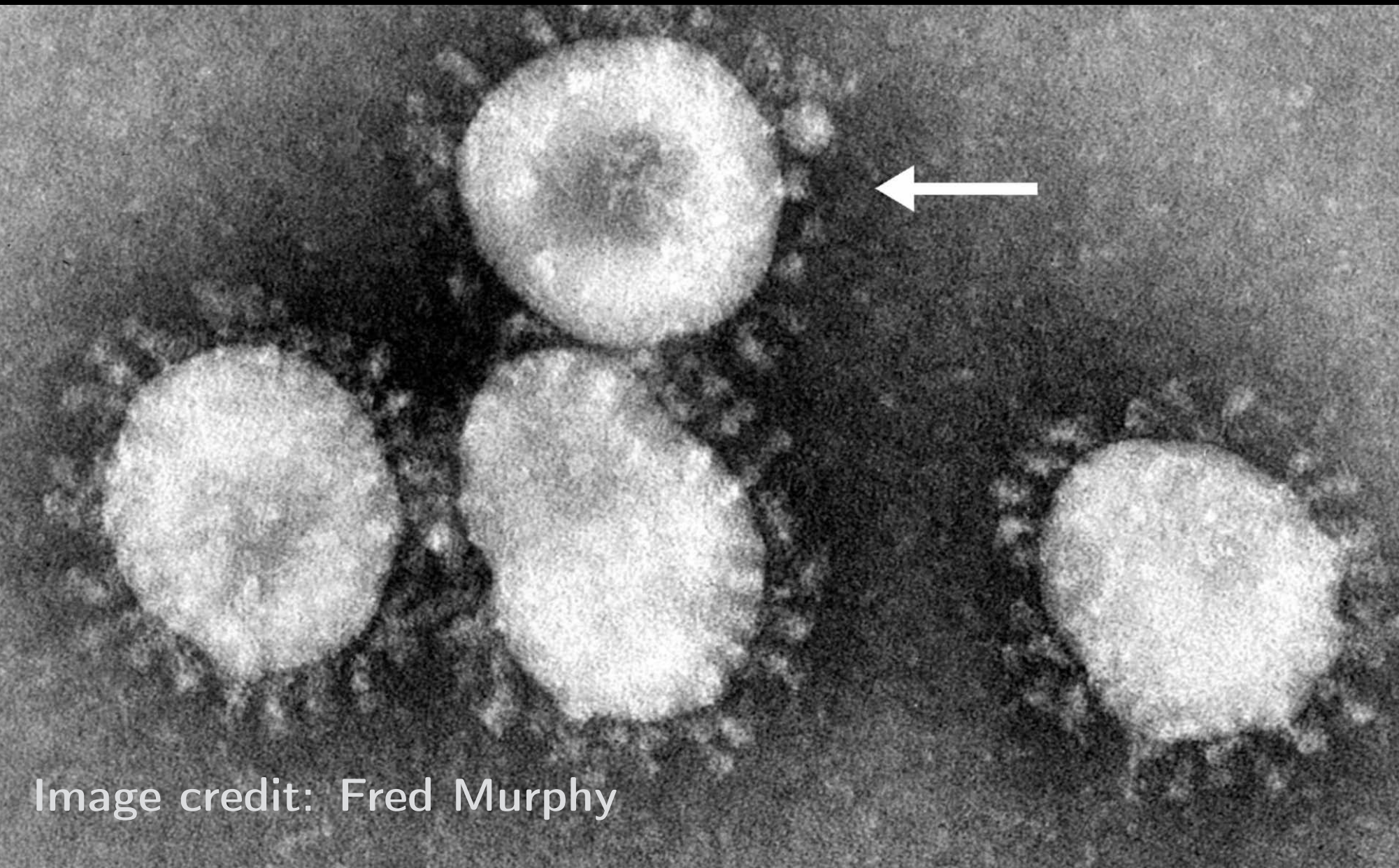
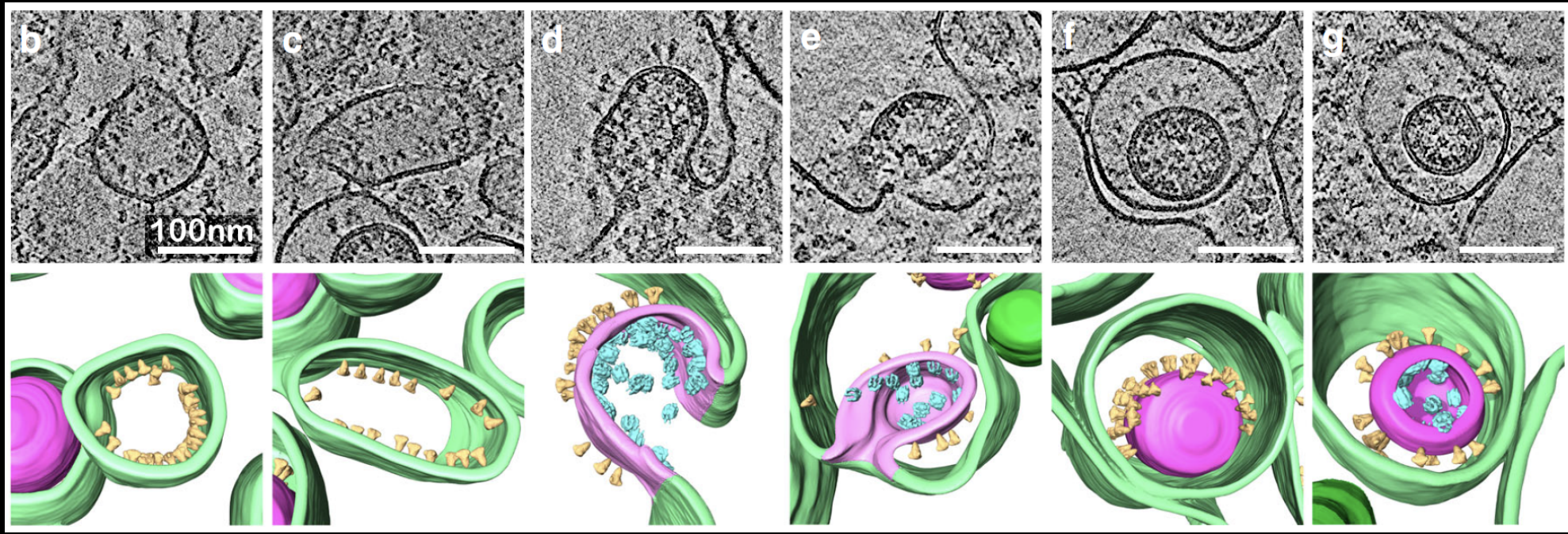


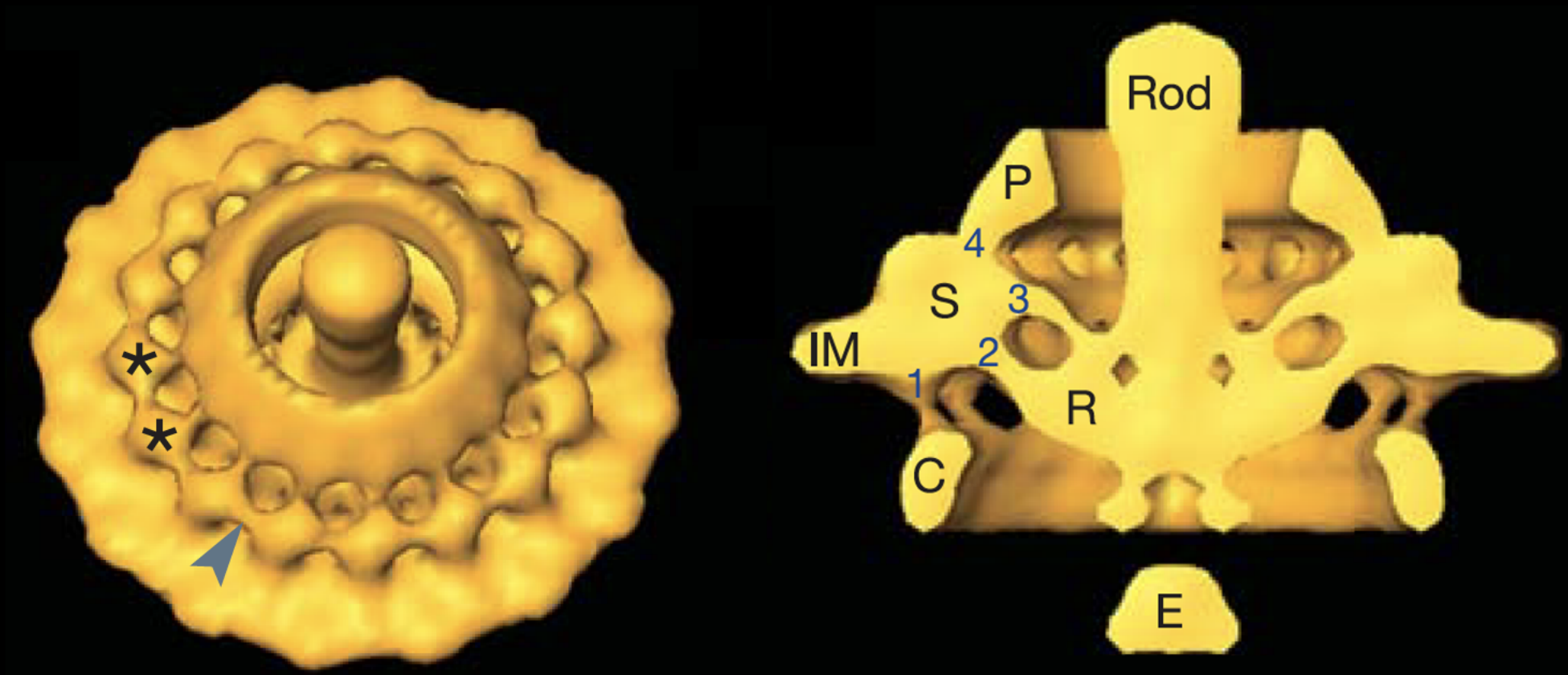
Image credit: Fred Murphy

Cryo-electron tomography reveals how a cell manufactures SARS-CoV-2 viruses



[Klein, Cortese, Winter, Wachsmuth-Melm, Neufeldt, Cerikan, Stanifer, Boulant, Bartenschlager & Chlanda 2021]

Electron transmission cryotomography reveals the swimming engine of *Treponema primitia* bacteria



[Murphy, Leadbetter & Jensen 2016]

Take-home message

Tomography is useful and can be done with many kinds of physics. However, the same reconstruction mathematics always works, regardless of the application!

Thank you for your attention!

←—Slime mold called Lycogala conicum